EXPECTATIONS AND FULL EMPLOYMENT:
HANSEN, SAMUELSON AND LANGE

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Abstract. From the outset, expectations were a central part of the first business cycles and early growth models. In the 1940s, a third line of research emerged which questioned the capacity of an economy to reach full-employment equilibrium. Starting with Alvin Hansen (1938) and culminating with Oskar Lange (1944), the crux of the debate evolved from the existence of full employment equilibrium to analysis of its stability, suggesting an increased role of expectations and finally challenging the economic system’s global stability. The present paper traces those debates through the contributions of Hansen, Paul Samuelson and Lange. Using archive materials, we show that while Samuelson’s analysis of instability remained implicit, his correspondence reveals that he encouraged Oskar Lange to examine it more carefully. Lange’s results are presented in his 1944 Monograph published by the Cowles Commission for Research in Economics. We point out that his contribution cannot be understood in isolation either from his exchanges with Samuelson or the way that Keynesian ideas were being interpreted in the United States. Finally, we emphasize the ambiguity of Samuelson’s view on instability and expectations.

Keywords: expectations, (in-)stability, full-employment, wages dynamics, Hansen, Samuelson, Lange.

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Introduction

Expectations have always been at the core of macroeconomics. They were a central part of pre-Keynesian economics, and an important dimension of the first business cycles models and growth theories. In the 1940s, alongside these works, and departing from the strict growth and/or cycles analytical frameworks, a new way of addressing dynamic issues emerged in the USA which questioned the capacity of an economy to reach full-employment equilibrium. The crux of the debate which started with Alvin Hansen (1938) and culminated with Oskar Lange (1944), evolved from the existence of a full employment equilibrium to analysis of its stability, suggesting an increased role of expectations and eventually challenging the economic system’s global stability.

The present paper tells the story of this third approach based on the arguments developed in the contributions by Hansen, Arthur Pigou, Paul Samuelson, and Lange. We draw on some of the correspondence between Samuelson and Lange to trace the progressive integration of expectations, and identify the important part played by Samuelson. His position on expectations was closely linked to his treatment of macroeconomic instability, although this is not always easy to grasp. To assess the scope of this third approach, it is worth briefly contrasting it to the way expectations were integrated into cycles models and growth theories.

Business cycle models developed along two lines depending on whether the cycles were generated exogenously or endogenously. On the basis of the distinction between “impulse problems and propagation problems” (Frisch, 1933), Keynesian ingredients were progressively integrated into the “exogenous” models. Expectations were introduced in two - albeit somewhat mechanical - ways; they were either used to derive investment functions (since expectations of future opportunities depending on current income) or were treated as shocks. Several economists such as Kalecki (1935), Kaldor (1940),

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3 Clearly, there was interest in expectations long before the emergence of macroeconomics, and it is usually considered that systematic analysis of expectations began with Thornton (1802).
4 They were also a central part of early pre-Keynesian economics. See Laidler (1999) for a careful analysis of the pre-Keynesian literature and the way Marshallian authors mostly paid attention to expectations.
5 Andvig (2006) explains that from the Stockholm School’s perspective, Frisch did not develop a convincing macrodynamics: “Frisch models were considered as static in the eyes of the Stockholm School, since economic agents, if they existed at all in his models, acted in a somewhat blind manner. The view that dynamics was more fruitful for macroeconomics than statics was thus not necessarily a shared view” (Ibid: 426). These comments on the rather mechanical role of expectations were based on that view. For a more detailed analysis of expectations theories provided by the Stockholm School see Hans-Michael Trautwein (2020) “Taking expectations seriously: A leitmotif in Stockholm School economics”.

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and Goodwin (1951) among many others, played a major role in developing an alternative line which proposed to explain cycles endogenously. Most of their models integrated expectations, for instance Kaldor’s (1940) non-linear investments and saving functions, and Goodwin’s (1951) use of a non-linear formulation of the accelerator interacting with a linear multiplier, later revived by Goodwin’s (1967) wage share – unemployment dynamics.

In the meantime, Harrod (1936, 1939) had taken a different direction. His aim was to show “that the trend of growth may itself generate forces making for oscillation” (Punzo 2009: 15), and he dedicated much effort to defining the line of steady advance, and identified the instability principle as the outcome of a disequilibrium between the effective and the expected accumulation realized by entrepreneurs. The way that expectations and income distribution were supposed to change during the adjustment process was essential to explain the behavior of what Harrod called the “warranted rate of growth”. Eventually, Harrod proposed a theory which assumed that cycles were generated endogenously along an exogenous trend. In that framework, expectations became fundamental elements of the system dynamics, conceived as a powerful propagation mechanism. Growth became progressively more central as Harrod (1939) highlighted the problem of the inequality between the effective, warranted, and natural rates of growth emphasizing that instability resulted from two problems: inequality between the effective and the warranted rates, and inequality between the warranted and the natural rates. Solow (1956), Kaldor (1957), and Pasinetti (1962) tackled the second problem with the aim to stabilize “the line of steady advance”. Solow’s model became dominant

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6 Hicks’s treatment of expectations deserves more specific attention since he clearly identified the issue in 1935 and then in 1939 proposed a systematic treatment of this question. However, in his 1950 book, Kaldor (1951: 839) notes that “On the subject of expectations, to which Mr. Hicks made such distinguished contributions in the past, the present work is remarkably silent”, insisting later in the same paper “It is very much to be hoped therefore that the distinguished author of Value and Capital will return again to that intriguing and still largely unexplored field [the role of expectations], which he seems for the present to have forsaken” (Ibid: 846). On Hicks’ treatment of expectations, see Lenfant (2020).

7 As rightly emphasized by Punzo (2009: 91): “The intuitive notion that the capitalist system is basically unstable belonged to the perception of reality of a broad intellectual milieu, spread between Oxford, Cambridge and probably the London School of Economics, and dominant in most Middle European circles. It was waiting to be made precise, which is precisely what Harrod was trying to do”.

8 Harrod’s reflections had emerged already in The Trade Cycle (1936) although in that book, the line of steady advance has no dynamics so that the cycle oscillates around this line (the future warranted rate of growth, Gw) and Harrod uses this steady growth path as a dynamic reference for his analytical framework and not yet as a medium which might formally link cycles and growth. The dynamics of Gw was not discussed until 1939.

9 See Besomi (2001) about the “mistaken attribution” of Harrod’s Essay to the theory of growth and, see Halsmayer and Hoover (2016) for an exploration of how Solow transformed the original Harrodian growth issue.
although Solow later acknowledged that his model solved only the second of Harrod’s issues. Also, in founding his model on a functional approach to investment and overlooking the difficulty associated with the introduction of an autonomous investment function, he clearly excludes entrepreneurial expectations. In the early 1960s, it became obvious that in order to ensure its relevance to economic policy, Solow’s growth model needed to prove the stability of the full-employment equilibrium path. Sen (1963) tackled this and in an unpublished paper was explicit: “it (Solow’s model) cannot do without an investment function” (Sen, 1964: 279). He added that “The difficulty is usually concealed by doing without an independent investment function in the growth models, and thereby by eliminating the influence of expectations. It is a dodge, and like all clever dodges it has its usefulness, but it is easy to outlive that” (1963: 280). In short, the neoclassical growth model provided stabilization at the expense of expectations and three decades later closed down the second approach of expectations and instability.12

Unlike these cycles and growth approaches, the Hansen-Samuelson-Lange connection attached great importance to expectations. Depending on how expectations were formed, this third way shed new light on the possibility of economic collapse. This line of research started when Hansen13 and Samuelson proposed a new understanding of the relation between the accelerator and the multiplier, inspired in 10

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10 In a letter addressed to Sen (mentioned in Sen [1970: 24 fn. 15]), Solow recognizes that his 1956 article was rather ambiguous and admits that “The idea was to trace full employment paths, no more.” So, here there is implicit acknowledgment (explicit in Solow [1988]) from Solow that his “answer” to Harrod’s “issue” generated some confusion by not clearly distinguishing the two instability problems which were at the core of the 1939 paper.

11 In 2009, in a private conversation with Muriel Dal Pont Legrand, Solow mentioned explicitly that part of the instability which deserves attention comes from the behavior of effective demand, and that Harrod saw this perfectly. Solow added that what was needed was an autonomous investment function, something which he considers we have never so far been able to provide.

12 See Assous and Dal Pont Legrand (2020) for a detailed investigation of the progressive exclusion of expectations from (neo-classical) growth theory.

13 It must be acknowledged that Keynes (1936) already questioned the role of expectations in equilibrium analysis. But, as emphasized by Rubin (2018: 67), if Keynes demonstrated how expectations can contribute to the determination of a given equilibrium position, he failed in analysing how expectations evolve from one period to the other. Hicks (1939) seemingly was confronted with the same problem and he recognised that the nature of his temporary equilibrium model - developed in 1939 - was ‘quasi-static’ and that it may have been because he “had [his] eyes fixed on Keynes” (1965:65). .
part by Harrod. In the context of the 1937-38 recession, Hansen used this model to interpret the role of government expenditures on full employment equilibrium and concluded that for a positive interest rate in the context of secular stagnation a full-employment equilibrium may not exist. Hansen’s analysis laid the foundations for Samuelson’s and Lange’s later work which led to the shifting of the debate on full-employment from Keynes to Hansen. While in 1941 Pigou was still questioning Keynes in *Employment and Equilibrium*, it is worth noting that it was Hansen who became the target in his famous 1943 “The Classical Stationary State” which was an attempt to prove the existence of full employment mechanisms. Samuelson - a former student of Hansen - had been behind the mathematical formulation of the accelerator-multiplier model and felt targeted by Pigou. He agreed with Pigou that under certain circumstances full employment equilibrium could exist. However, Samuelson moved the problem away from the issue of existence and introduced the issue of stability: by going back to his early 1940s methodology, we can show that he believed it was possible that full employment might be unstable because of expectations. While his analysis of instability remained implicit, his correspondence reveals that he encouraged Oskar Lange to examine it more carefully, and Lange’s results are presented in his 1944 Cowles Commission Monograph.

I/ Expectations in the accelerator-multiplier model

Nothing prepared Alvin Hansen for his key role in the “Keynesian Revolution”. Hansen, already in the 1930s a respected business cycle economist, left the University of Minnesota in 1937 to go to Harvard University. It was there that faced with the 1937-38 recession and the spread of Keynesian ideas, he

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14 See for instance Besomi (2003) who perfectly explains Harrod’s influence on Samuelson as well as the differences between the two 1939 vintage models. It is interesting to note that the transformation achieved by Samuelson in converting Harrod’s multiplier-accelerator analysis into “two independent lagged equations, with constant coefficient but feeding one equation with the result of the other … represents a process similar to the one described by Harrod, but one which acts mechanically, without room for expectations” (ibid: 317).
15 See Dockes (2015) or Backhouse and Boianovsky (2016a) for a detailed presentation of Hansen’s seminal contribution to the concept of “secular stagnation”.
16 Hansen’s connection to Keynes *General Theory* is well-known; however, it should not be overemphasized since as Backhouse and Boianovsky (2016b) remind us “Hansen’s secular stagnation thesis was derived largely independently of Keynes’s *General Theory* (1936). He had obviously read the book, reviewing it twice and coining the expression ‘under-employment equilibrium’ to describe Keynesian theory (Hansen 1936), but he did not consider Keynes’s theory, which he perceived to be too static, to be the same as his own.” (ibid, p. 152).
17 See Backhouse (2017) on the myths related to Hansen’s “conversion” to Keynesian economics and Laidler (1999) on the fabrication of the “Keynesian Revolution”.
assumed the leadership in a revolution – a revolution in economic theory. The 1937-38 crisis sparked different theoretical interpretations of the role of the state in the economy; Hansen’s interpretation proved to be one of the most important. His view was embedded in a model of the interaction between two relatively well-known and already central economic mechanisms: the accelerator and the multiplier.\textsuperscript{18}Hansen’s starting point was the shock caused by the brutal 1937-38 recession which occurred after a four-year period of recovery from the 1929-1933 great depression. Looking back on this period of recovery he concluded that it was a “consumption recovery” driven mainly by federal expenditure on social relief and the automobile industry (Hansen, 1938: 276). However, following a period of boom this fiscal stimulus ended in 1936, and when private investment failed to replace this government expenditure there was a sudden collapse of income, production, and employment.

To explain these fluctuations, Hansen sought to show that a constant and continuous level of government expenditure was needed to maintain income at a new and higher level (Hansen, 1938: 280). He focused on the accelerator and the multiplier and, tried to work out their interactions. He presented these mechanisms as the two directions of the relation between consumption and investment: the accelerator linked the changes in consumption to new investment, and the multiplier linked changes in investment to consumption through higher national income (with the relation between income and consumption remaining constant at different levels of income). New investment depends on changes in consumption, if consumption ceases to increase, induced investment falls back to nil, and in the absence of any autonomous investment, the gap between full employment income and consumption reduces income also to zero. Thus, the first consequence of reducing expenditure geared to consumption is to diminish investment.

Yet Hansen’s analysis remained rather vague, and he worked out those interactions in a literary manner in his published analysis of the crisis (Hansen, 1938). Nevertheless, it seems that he had established the basic equations of a model which he subsequently presented to his students in his famous Harvard seminar.\textsuperscript{19} Samuelson seized this opportunity to apply his mathematical skills to the study of a dynamic

\textsuperscript{18} The accelerator was developed at the end of the 1920s, in particular in the work of A. Aftalion. In the United States, it was introduced in a debate between J. M. Clark and Frisch conducted in the \textit{Journal of Political Economy}, in 1931-32. The multiplier came from the works of R. Kahn and J. M. Keynes during the first half of the 1930s. During the second part of that decade, few economists most notably Harrod, Hansen and Samuelson, tried to work out their interrelations.

\textsuperscript{19} Samuelson (1940, 1959, 1988) gives much credit to Hansen for the model. See also Salant (1976) on the Fiscal Policy seminar led by Hansen and John H. Williams, where Hansen presented the model for the first time. The importance of this
system, and published a paper (Samuelson, 1939a) in which he worked out the different possible trajectories, and their stability or instability. His model comprised three equations: the “relation”\(^{20}\) between current investment \( (I_t) \) and current consumption \( (C_t) \): \( I_t = \beta (C_t - C_{t-1}) \), the consumption function \( C_t = \alpha Y_{t-1} \), and an equilibrium condition: \( Y_t = I_t + C_t + A \) where \( A \) is the amount of autonomous investment and \( Y_t \) is the output.\(^{21}\) This model includes expectations which can be either endogenous to the system state or exogenous, and are associated with the corresponding different effects. However, it should be noted that expectations are not the main focus of the model. In Hansen’s work, they appear to explain secular stagnation, mainly as a result of exogenous factors but not completely endogenous to the economic system. Samuelson’s exposition leaves room for both types of expectations but he stops short of clarifying their roles. He later assigned them different roles when he was focused mainly on stability in relation to his reading of Pigou’s 1941 book (see section II), whereas Hansen was interested initially in proving the non-existence of a full employment equilibrium.

In his second article on the accelerator-multiplier model, Samuelson is more explicit about the part played by expectations: he describes the constant amount of investment in \( A \) as “spontaneous, anticipatory investment,” a feature of “any healthy society” (Samuelson, 1939b: 790). Because it is unrelated to changes in income, it is an exogenous quantity determined by several factors including expectations as he makes clear in another article on those issues written around the same time:

> The amount of net investment must be regarded as dependent on dynamic factors of economic progress such as the amount of yet undeveloped innovations, trend of population, past net investment, as well as upon the shifting state of confidence and expectations. (Samuelson, 1940: 493).

Endogenous expectations are reflected in the acceleration coefficient \( (\beta) \). In that case, they have a direct impact on the induced investment \( I_t \). He makes this clear when slightly later in the article, he examines in particular the case of “pure pump-priming”:

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\(^{20}\) A clear reference to Harrod, who is discussed in Samuelson (1939b).

\(^{21}\) Samuelson only considers government expenditures as a component of \( A \) in his first article; in the second article he generalizes his results to all autonomous investment in his second article.
Some attribute the insufficiency of net investment to the fact that the slump in business has produced a perverse state of pessimistic expectations which inhibits the entrepreneur and prevents him from performing bold acts of investment. They argue that the introduction of known innovations is a function of the current state of activity of the system. (Samuelson, 1940: 502-503).

In a later piece, he clarifies the difference between endogenous and exogenous expectations in an attempt to explain the debates around the multiplier which were sparked largely by the accelerator-multiplier model. In the opening pages on the stability of the multiplier when the marginal propensity to invest is introduced along with the marginal propensity to consume, Samuelson (1942b: 579) underlines that a “rise in anticipations which shifts the investment schedule upwards is not to be confused with the slope of an instantaneous schedule”. The importance of this distinction lies in the conclusion that Samuelson and then Hansen draw on the model: an analysis of the difference equation arising from the above system of three equations indeed shows that the equilibrium level of the system is determined only by the marginal propensity to consume and by autonomous investment, according to the familiar multiplier formula: $A \cdot \frac{1}{1-\alpha}$. On the other hand, the acceleration coefficient affects only the system trajectory, and generally speaking as this coefficient increases the system becomes unstable.\textsuperscript{22} This was noted explicitly by Hansen in his 1941 book where he integrated Samuelson’s analysis saying that: “the addition of the Acceleration Principle to the Multiplier Principle does not, in this case, affect the ultimate level of the national income but only the intervening path through which the income moves” (Hansen, 1941: 278).

From this, it follows that exogenous expectations can raise or lower the income level through the amount of autonomous investment, while endogenous expectations can have a potentially destabilizing effect via the acceleration coefficient. However, at this point, neither Hansen nor Samuelson linked the value of $\beta$ to either wages or prices which remained fixed. Hansen himself was more interested in the determinants of autonomous investment which is where his idea of secular stagnation comes into play. Indeed, while business expectations about investment decisions are dependent in large part on innovations and potential outlets, Hansen thought that the most recent

\textsuperscript{22} The reader may like to consult Samuelson (1939a) for a proof. The diagram on p. 78 of Samuelson’s article shows clearly that for increasing values of the acceleration coefficient, the propensity to consume needs to become smaller and smaller for the system to remain stable.
wave of innovations was dying, and in contrast to the 19th century the American border was closed, westward expansion had been halted, and population growth had already declined seriously (Hansen, 1938: 313). These factors maintained a low level of expectations, inducing a low level of private autonomous investment and a tapering off of income: he called this secular stagnation, a stagnation of income at a level below full employment (Hansen, 1938; 1939).

At the end of the 1930s, following a decade of depression, meager recovery, and recession, Hansen’s outlook regarding future investment opportunities was bleak which explains why he turned to the role of the state to maintain income growth: “Viewed against this background, governmental expenditures take on a new significance. From being purely a cyclical compensatory device, designed to stimulate consumption, public expenditures may come to be used increasingly as a means of directing the flow of savings into real investment” (Hansen, 1938: 289). For Hansen, the consequence of low levels of profit, income, and expectations was that a positive interest rate corresponding to a situation of full employment did not exist. The lack of investment outlets meant that the economy was stuck in an underemployment equilibrium. Although earlier writers had foreseen the decrease in investment opportunities, they “did not raise the question whether such stability would be achieved at a full-employment and full-income level” (Hansen, 1939: 4). For Hansen, this meant that in the context of secular stagnation, coordination through markets was unable to induce a situation of full employment, a conclusion provided in his 1941 book, Fiscal Policy and Business Cycles:

The classicals were quite right when they argued that without technological progress the price system, including the rate of interest, would progressively drive the economy to the point at which there would be no net investment. They were wrong in assuming that the price system could also ensure a propensity to consume compatible with this investment situation so as to provide full employment. (Hansen, 1941: 288)

The idea that the classical stationary state was not necessarily accompanied by full employment was an essential point because it offered the most compelling argument in favor of government expenditure to achieve full employment by supplementing the gap with private autonomous investment. However, Hansen failed to explain the influence of falling wages and prices on expectations, the rate of interest, and the idea of stagnation at a level of income short of full employment was the focus of his opponents’ attacks in the 1940s. The notion of stagnation raised concerns among economists because it questioned the self-adjusting properties of the economic system. Pigou in particular saw Hansen as a major
proponent of Keynesian ideas, and attacked him in Pigou (1943) by suggesting that the price system included mechanisms which could return the economy to a position of full employment equilibrium.

Hansen did not respond directly to these claims. He did not need to since the 1938 crisis and the war had increased his influence on the political elites in Washington who progressively adopted his line of reasoning about government expenditure. On the other hand, Samuelson was not entirely comfortable with the idea of the non-existence of a full employment equilibrium in the presence of a positive interest rate. At the beginning of the 1940s, Samuelson embarked on a different road from Hansen, and redefined analysis of income determination as an equilibrium rather than a process in which the issue of stability takes center stage.

II/ Expectations and comparative statics

Can unemployment be prevented by lowering wages and costs? Samuelson did not tackle that question head on, and never for instance - on the basis of a model - considered how an economy might react to falling wages. This omission can be understood only in a comparative statics framework: for Samuelson, it could only involve stable equilibria, and thus the problem of stability was “intimately tied up with the problem of deriving fruitful theorems in comparative statics” (Samuelson, 1941b: 98).

According to Samuelson, the task of comparative statics was “to show the determination of the equilibrium values of given variables (unknowns) under postulated conditions (functional relationships) with various data (parameters) being specified” (Samuelson, 1941b: 97). Samuelson stressed that two alternative conditions are required for the comparative statics method to apply: the equilibrium position should be a maximum (for “single economic units“), or stable (in the case of multiple economic units). He pointed out that: “The problem of stability of equilibrium cannot be discussed except with reference to dynamic considerations ... . In order for the comparative-statics analysis to yield fruitful results, we must first develop a theory of dynamics” (Samuelson, 1941b: 102). So, one possible interpretation of why Samuelson did not conduct static comparative exercises in relation to wages is

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23 Pigou was not the only one at that time to consider Hansen the main North American Keynesian figure. Kaldor (1939) saw Hansen not only as “the leading authority on business-cycle problems in the United-States” but also as the “nearest to Mr. Keynes”, regarding “fluctuations in the demand of investment as the fundamental cause of cyclical fluctuations” and concern over “the danger of chronic stagnation”. (Kaldor 1939: 91).

24 On Hansen’s influence in Washington, see Backhouse (2017, chap. 19).
that he had no doubt that such exercises were meaningless due to the instability of the full employment equilibrium.

Because Samuelson recognized that income theories and the existence of a full employment equilibrium issue involved aggregated variables and not “single economic units”, he stressed the importance of basing their analysis on the stability properties which are the necessary condition for comparative statics at this “macro” level. In an unpublished paper (Samuelson, 1942a), he agrees with Hansen that assuming that consumption and saving are related to income, a system with zero net private investment cannot maintain a level of national income and employment higher than the income break-even point at which saving is zero. Consequently, full employment implies that the investment curves are sufficient to offset the saving which would have been made at that level. Otherwise, the level of income would fall as businesses curtail their spending in response to inventory accumulations.

Is net private investment sufficient to assure full employment equilibrium? Samuelson is skeptical about this. He believed that as the stock of capital grew through the accumulation of positive net investment, the increased income ultimately would result in zero net investment. In addition, assuming now a declining marginal propensity to invest - a lower \( \beta \) in his 1939 model framework - he concluded that the possibility that saving would exceed investment at full employment would likely lead the economy to a less favorable but still stable equilibrium where the level of income was associated with underemployment. Only active macroeconomic policies such as public spending or tax policies, are able to raise the level of national income to the neighborhood of full employment, the stability of equilibrium ensuring - in accordance with the correspondence principle - that such policies could be implemented.

Unlike Samuelson, Pigou (1941) considered that full employment could be reached through adjustments to money wages. Pigou used an aggregate model integrating a Keynesian consumption function but with two arguments: income and the interest rate. He argued that deflation would cause a fall in the interest rate which would reduce saving and increase investment, and that the bottom line was the existence of a positive rate of interest compatible with full employment. Samuelson challenged this view: "I fear that few economists will follow him (Pigou) in believing that under modern conditions there exists a positive rate of interest at which saving out of full employment income would be zero" (Samuelson, 1941a: 549). Like Hansen, he based his argument on the idea that for both technical and psychological reasons investment was very inelastic with respect to interest rate changes and, was likely to fall short of saving for any positive rate of interest.
More importantly, Samuelson stressed that even if wages flexibility were able to ensure the existence of a full employment equilibrium under certain circumstances, this equilibrium would be unstable. He believed in the possibility that the psychological factor of expectations generated by price changes, and generating changes in the same direction, would lead to an unstable system. Following his methodology, “the question is begged as to whether a given change is followed by a ‘once-for-all’ adjustment, or a never-ending one” (Samuelson, 1941a: 552). That instability was crucially important for Samuelson since it prevented any comparative statics exercise or application of the correspondence principle. According to Samuelson, it is extremely dangerous to propose adjustments to a variable for which the equilibrium is not stable since that would result in the collapse of the whole economy. This definitely explains why Samuelson did not provide a comparative statics exercise based on a change in money wages.25

The problem of instability is apparent also in his correspondence with Lange. In March 1942, these two economists exchanged a couple of letters on the problem of the “optimum” propensity to consume,26 and on the role of flexible prices. Lange explained to Samuelson that he was only now beginning to see the importance for the “new classical theory” of flexible prices and wages:27

I am inclined to be more charitable to the new classical theory than I was before or than you appear to be. ... As I see it now when wages & prices of capital goods are flexible their fall produces excess cash balances in addition to the excess produced by the fall in real income which strengthens the pressure on interest rates.

A few days later,28 Samuelson wrote that he had yet to be convinced about the soundness of flexible prices:

As you say, many of them [neo-classicals] would insist upon the efficacy of wage reductions but rarely in connection with their business cycle studies and only as a reflection of their implicit belief that some low enough price will clear every market. Moreover, I would not attach the

25 Samuelson argued also that such exercises were meaningless in economies where prices and money wages were fixed. From 1940, he had claimed that there was “good reason to doubt that much downward flexibility exists in an economy such as our own” (Samuelson 1940: 496) and he maintained that position, see for instance Boianovsky (2020) and Samuelson interview in Colander and Landreth (1996).

26 In 1938, Lange wrote a paper entitled “The Rate of interest and the Optimum Propensity to Consume” which gave rise to a correspondence between him and Samuelson. It is with reference to Lange’s model that Samuelson based the second part of his 1941 Econometrica paper.

27 Lange to Samuelson, March 2nd 1942, Paul Samuelson Papers, Duke University.

28 Samuelson to Lange, March 11th 1942, Paul Samuelson Papers, Duke University.
importance that you do to the stimulation of consumption but the fact that a lowering of wages and prices increases the real wealth involved in relatively unchanged cash balances. Any such effect could only be a temporary one. Both casual observation and statistics suggest that consumption depends more upon income than upon wealth. A rapid enough increase in real wealth will stimulate consumption but it takes ever-continuing wage reductions and price reductions to keep consumption up. It is my belief that no wage level, however low, if long maintained can permanently stimulate consumption.

At the end of his letter, Samuelson insists that the “application of these plans would lead to the introduction of hyper-deflation with disastrous consequences to our economic and political system. I shall be very sorry, therefore, if in your monograph you give way on these points” (ibid). Influenced undoubtedly by this letter, Lange integrated the destabilizing role of expectations into his own work.

### III/ Expectations and instability

Lange was convinced that improvements in macroeconomics would come primarily from careful attention to the interdependencies between markets. His 1944 Cowles Commission Monograph was the result of a strategy based on combining Hicks’s *Value and Capital* (1939) - a volume he considered the “most up-to-date formulation of the theory of general economic equilibrium” (Lange, 1944: viii) – with Samuelson’s stability analysis. Lange argued that Keynes’s proposition that under certain conditions, “changes in money wages have no effect upon employment but influence only the level of product prices” (ibid: 1) could be based on new foundations and that this would challenge the “traditional economic doctrine” that unemployment might result from downwardly rigid prices and nominal wages. The bottom line was that in most circumstances, a full employment equilibrium – taking for granted its existence and uniqueness – would prove unstable and not securable through wage flexibility. However, Lange’s reference to Hicks might be misleading. The dynamic aspects of his

29 The date on the first page of the Monograph is 1944, the copyright date is 1945. In the rest of this article we reference this work as Lange (1944) because he had completed it well before 1945.

30 In his 1938 model, Lange was already claiming on the basis of an aggregate model that an important relationship between Walrasian general equilibrium and Keynesian economics existed. That relationship was recently examined by Rubin (2016), and the influence of Keynesian theory on Walrasian theory through Samuelson’s early works was addressed by Hands (2012).

31 “Some authors,” adds Lange, “even maintain that the relationship is the reverse of what is taught by traditional doctrine, i.e., that a rise in money wage rates increases and a fall in money wage rate decreases employment.” (Lange 1944: 2).
approach are borrowed from Samuelson who did not study a sequence of temporary equilibria but rather the process of convergence to a unique equilibrium; the proximity between Lange and Samuelson on this point was underlined by Hicks himself in his review of Lange’s monograph (Hicks, 1946).

In a short opening chapter on the theory of partial equilibrium, Lange argues that each individual assumes that his own decisions are so insignificant that they do not affect other individuals. In that context, if there is an excess supply of a factor of production in one market then provided the price of that factor is flexible, equilibrium is restored with no impact on other markets. This occurs through two main channels: a “substitution effect” and an “expansion effect”. A fall in the factor price induces a substitution of this factor by other factors. In turn, this lowers the marginal cost and is translated into a rise in output. For instance, if there is excess supply in the labor market then as money wages fall firms will switch to labor intensive production processes (substitution effect). At the same time, as the marginal cost curve shifts downwards, output will expand (expansion effect). Ultimately, if money wages reduce sufficiently, any excess supply will be absorbed.

Lange suggested that to move the argument in a macroeconomic direction required shifting from a partial-equilibrium to a general-equilibrium analysis. This means in particular, being prepared to take account of the repercussions of a change in the price of one factor on the remaining prices. For a given excess market demand, general equilibrium requires only the assumption of Walras’s law and zero degree homogeneity of the excess demand function as explained in detail in the mathematical Appendix. In a monetary economy, Lange explained that any demand for goods implies a supply of

\[\text{\ foreign\ currency\ demand \ for \ home\ goods}\]

\[\text{\ domestic\ currency\ demand \ for \ foreign\ goods}\]

\[\text{\ domestic\ money\ demand \ for \ foreign\ money}\]

Under Samuelson’s influence, Lange departed from Hicks’s perspective. In the second edition of Value and Capital Hicks acknowledges that Lange considers “the process analysis investigates the convergence (or lack of convergence) of the system towards a position which equilibrium analysis determines” (Hicks 1946: 242). Nevertheless, Lange also recognized his indebtedness to Hicks and his Value and Capital, by placing the elasticity of expectations as an element of fundamental importance for the stability analysis of equilibrium (Lange 1944: viii).

“Price flexibility is defined as follows. The price of a good is said to be flexible if it falls whenever there is excess supply of it and rises whenever there is excess demand for the good. In the opposite case, the price is said to be negatively flexible. The price is said to be inflexible, or rigid, if excess supply or excess demand fails to affect it.” (Lange, 1944: 2)

In the appendix, Lange makes clear that these two properties come from the assumption about the behavior of the underlying agents and could be derived from rational choices. See Hands (2012) for a detailed account of that aspect.

Formally, as shown in the appendix of the Monograph, necessary and sufficient conditions for local stability follow from the first order approximation to a system of n independent excess-demand functions. Along the lines of Samuelson's thinking, Lange introduced the concept of “partial stability of different order and rank” to address cases in which some
money that can be exchanged for it, while any supply of goods implies a corresponding demand for money. Logically, if demand equals supply for each good in the economy, the demand for cash balances will be equal to the quantity of money in existence. Consider again the possibility of an excess supply of labor. Supposing a willingness to assume temporarily that expectations are static, i.e. “prices are expected to continue in the future”, restoration of equilibrium will depend on the strength and direction of what Lange calls a “monetary effect” resulting from the two effects initially derived in a partial-equilibrium context. The discussion centers on three cases. When overall deflation causes a less than proportional change in the excess demand for money the original excess supply of labor is eliminated. In that case, Lange shows that the monetary effect is positive. Conversely, if the monetary effect is negative, a proportional change in all nominal prices causes a more than proportional change in the excess demand for money. Deflation becomes self-aggravating and full employment is not restored. Finally, if all prices change in the same proportion, substitution and expansion effects are no longer operative. Output remains unchanged and the monetary effect is neutral.

The two last cases open the possibility that an initial burst of excess labor supply may not be eliminated automatically by price flexibility, although as long as expectations are static and the quantity of money remains constant, these two cases are not possible:

the condition that the real quantity of money be increasing as prices fall and decreasing as prices rise is automatically satisfied when the nominal quantity of money in the economy is constant. In this case, the monetary effect is always positive and flexibility of factor prices automatically maintains or restores full employment and prevents or absorbs excess demand for factor of production. (Lange, 1944: 14).

In a footnote, Lange refers to Pigou’s famous 1943 paper on “The Classical Stationary State” saying he became aware of it only after his manuscript was finished.36 Pigou’s argument, though expressed in an aggregate framework, is similar to the one raised by Lange. Due to the drop in prices, the real stock of money necessarily increases and eventually induces agents to absorb any positive excess demand for goods. With the aim of reviving Keynes’s line of thinking, Lange’s challenge was to identify the prices in the economy may be fixed. From this concept, new stability conditions can be derived: “The system is said to be partially stable of order m (m ≤ n) if [the excess demand condition] is satisfied when only m other prices are adjusted and the remaining prices are kept constant” (Lange, 1944: 93).

36 Thus, Pigou’s analysis of the argument was based on a real balance effect which is assumed to have a direct impact on consumption and is supposed to be effective even for a constant rate of interest.
conditions where a full employment equilibrium might be unstable, and hence to point to the factors likely to cancel out the positive real balance effect. Lange thought this was justification for the full integration of expectations into general equilibrium theory, and that once this was done, the stability conditions could be derived easily from Samuelson’s dynamic analysis.

Lange argued that the possibility of adverse speculative effects with falling prices could result from an “intertemporal substitution effect” based on the observation - as much as on the reading of Hicks’s works - that deflation may give agents an incentive to delay current expenditure in order to benefit from a lower future price. In that context, “the monetary effect” may prove inoperative to ensure equilibrium stability. For instance, if the elasticity of expectations defined as the “ratio of the proportional increment of the expected price to the proportional increment in the current price” is more than unity, wage adjustments resulting from the difference between labor supply and demand could cause a downward spiral of prices and output. So, while static expectations ensure the stability of the full employment equilibrium for any given quantity of money, elastic expectations do not. In addition, Lange’s model directs attention to the role and nature of the interdependencies between the good and bond markets and the direct impacts of their dynamics. As real cash balances increase, demand for bonds increases, and because bonds are debts whose face value and interest payments are “fixed by contract”, their increased price materializes in a fall in the current interest rate which stimulates investment. Of course, Lange argues that this effect (known as the Keynes effect) vanishes when investment is inelastic to the interest rate, and becomes null in the liquidity trap situation. In this case, instability can be traced to the fact that price adjustments engender pessimistic expectations about future prices which are not offset by the fall in the rate of interest.

What does this deflation process mean for unemployment? As Lange emphasizes in his opening remarks in chapter 2, as soon as “underemployment is defined as an excess supply of a factor of production” (ibid: 6), this necessarily means that it has a counterpart in excess demand in at least one market. Most importantly, this means that along the whole process of adjustment, the gap between

37 Lange notes that static expectations represent a special case of expectations of unit elasticity.

38 For these two reasons, Lange concludes, “A positive monetary effect is much less likely to assure automatic restoration of equilibrium through price flexibility when it implies direct substitution between money and securities” (Lange 1944, p. 18)
the supply and demand curves will keep growing.\textsuperscript{39} It should be noted that while they were much more central in the text, expectations remain somewhat hidden in the appendix. Much like Hansen and Samuelson who put expectations in $\beta$ or $A$, Lange seems to include them in the parameters of the excess demand functions. In Hicks’s analysis, each temporary equilibrium is predetermined by a given state of expectations. Once these expectations are known, a weak equilibrium is reached. A sequence of temporary equilibria may be established after expectations are revised. Depending on expectations elasticity, this sequence will lead the economy to a stationary equilibrium or not. This means that stationary equilibrium stability depends crucially on the elasticity of expectations. However, Lange favored a different approach where it no longer made sense to predetermine the equilibrium based on the state of the expectations because expectations are revised throughout the adjustment process.

**Conclusion**

In this paper we focus on a specific episode to show that the increasing lack of interest in expectations was a result of the way in which Samuelson tackled the problem of wage and price flexibility. He chose repeatedly in his published work not to provide a detailed and thorough study of the possible effects of a shock on money wages. This then raises an important question to which there seems not to be a single clear answer: why did Samuelson refuse to address explicitly the problem of the impact of wage and price flexibility on full employment stability?

We can propose various – practical, empirical and political – explanations. The simplest explanation is a practical one – that Samuelson did not have sufficient time to address this issue: war had broken out, he had a job in Washington, he was a young professor at MIT, and he lacked the time to develop and publish his ideas.\textsuperscript{40} Alternatively, there are some empirical breaks in the idea that wage flexibility might be the mechanism allowing a return to full employment: wages were evidently not flexible and Samuelson was often concerned by the description of the real world. He made this clear on several occasions including in Samuelson (1992) and in an interview in Colander and Landreth (1996). Tobin

\textsuperscript{39} The same point was made by James Meade in “A Simplified Model of Mr. Keynes’s System”: “If we suppose that the money wage-rate would fall so long as any labor were unemployed, the system cannot be in equilibrium without full employment” (Meade 1937, fn. 2, p. 99).

\textsuperscript{40} See also Backhouse (2017: 370-380) on this explanation, and the multiple activities in which Samuelson was involved at the time.
(1983) supports this explanation. There is a third perhaps political explanation: the general political atmosphere at this time was not favorable to anti-capitalist ideas. During his time in Washington, in 1943 Samuelson witnessed first-hand the shutting down by the Conservative Congress of the National Resources Planning Board for which he was an advisor, (Backhouse, 2017: 402sq.). Thus, advocating for a manifest failure in market economies would likely have been career suicide for a budding young economist already viewed with some suspicion because of his Keynesian textbook (Giraud, 2014).

Lange was not concerned about sparing the feelings of big business and the advocates of a free market. However, this research direction became an increasingly more precarious pursuit for him and made it impossible to establish a school of thought. During the second half of the 1940s, he became involved with the new communist Polish government, and subsequently resigned from his professorship at the University of Chicago. At the Cowles Commission, where he did most of his work in the United States and which was then housed in the University of Chicago, his thinking was subject to increasing attacks from other economists. In particular, Milton Friedman who had just returned to Chicago was a relentless critic of the Cowles Commission approach to economics, and ultimately was one of the main reasons why the Commission moved to Yale (Dimand and Rivot, 2020). His 1946 critique of Lange’s monograph and theoretical approach was particularly devastating and may have played a role in extinguishing the idea of global instability.

Might this line of research be relevant today? We should highlight that recent renewed interest in stagnation theories is being accompanied by increased interest in the idea of societal “collapse” among those concerned by environmental problems and (more recently) global pandemics. Both crises wreak havoc on expectations and wages causing the latter not just to fall but, in many cases, to disappear completely. While these two aspects of stagnation and collapse have yet to be reunited with the notion of stability, revisiting Hansen’s, Samuelson’s, and Lange’s ideas on these matters could provide a new perspective on expectations and instability of the economy.
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