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The Unilateral Accident Model under a Constrained Cournot-Nash Duopoly

(Preliminary version)

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Summary: This paper extends the basic unilateral accident model to allow for Cournot competition. Two firms compete with production input and prevention as strategic variables under asymmetric capacity constraints. We find that liability regimes exert a crucial influence on the equilibrium price and outputs. Strict liability leads to higher output and higher risk compared to negligence. We also study the conditions under which both regimes converge.

Key Words: Tort Law, Strict Liability, Negligence Rule, Imperfect Competition, Oligopoly, Cournot Competition

JEL Classification: D43; L13; L52; K13

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1. Introduction

In modern economies, mass production generates industry and energy catastrophes as undesirable byproducts. Due to natural hazard (the hand of God), some disasters are unavoidable; however, most of the others can be explained by human negligence or wrongdoing. For instance, on April 24, 2013, near Dhaka, Bangladesh, a garment factory (the Rana Plaza) collapsed without warning, killing more than a thousand workers and injuring two thousand. Several well-known Western trade companies used this workspace as subcontractors. They expected benefits from delocalizing in Bangladesh through potential quadruple dumping i) low wages, ii) weak payroll taxes, iii) weak corporate taxes, and iv) deficient environmental regulation. Judicial investigations showed that these firms’ negligible investments in safety had triggered this disaster.

The breakdown of the Deepwater Horizon oil rig leased by British Petroleum (BP) (April 20, 2010 in the Gulf of Mexico) is another example. Although the number of killed and injured by this event was comparatively small, it caused extensive pollution of U.S. territorial waters. BP had tried to exploit rich pockets of oil in the deepest offshore well ever dug in the Gulf of Mexico. The subsequent legal investigations revealed significant deficiencies in securing the project. In France, a salmonella scandal at the French dairy group Lactalis at the end of 2017 affected 83 countries. The French authorities required Lactalis to recall almost 12 million infected boxes. Nevertheless, between 2015 and 2017 some 200 babies suffered from the infection. Since Lactalis’s liability appears obvious, both the victims’ parents and the French authorities currently are working to bring the corporate to Justice. In these examples, huge underinvestment in safety caused major accidents involving the firms in both criminal and civil liability.

Since the end of the 19th century, the industrial revolution has facilitated the expansion of tort law to compensate harm and provide reparations to third parties. In reviving the Law and Economic tradition, Ronald Coase (1960) and Guido Calabresi (1961) showed that effective tort law leads potential injurers to invest in prevention to avoid out of proportion reparations and compensation. As a result, due to their capacity and financial constraints, firm managers are constantly arbitraging resources between production and prevention activities. Thus, it is legitimate to question whether liability

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regimes affect competition, and whether the effects if any change according to the specificities of the liability regime, and by extension whether under competition, a specific liability regime is more efficient than another? The present article addresses these questions.

We depart from the basic accident model’s assumptions as defined by Calabresi (1970), Brown (1973) and especially Shavell (1980, 1982, 1987, 2004) by introducing competition between two injurers under different liability regimes, namely strict liability and negligence. We develop a constrained Cournot-Nash duopoly model for a perfectly substitutable good with production inputs and prevention levels as strategic variables. A previously rational and benevolent regulator enforces a given liability regime.⁵ We follow Pu-yan Nie and You-hua Chen (2012) who consider constrained inputs but in contrast to their paper the firms in our model compete under asymmetric capacity constraints. These constraints on each firm’s strategy set imply that it is not straightforward to establish a correspondence between the model solutions and the social first best of the standard unilateral accident model in which a representative firm disposes of inexhaustible resources when determining its optimal level of prevention.

There is a large literature that compares competitive market structures using the accident model under the product liability motive, and which has been reviewed by Daughety and Reinganum (2013) and Geistfeld (2009). Pioneering authors compared the efficiency of different market structures under product liability (see e.g. Epple and Raviv, 1978). However, the interplay between industrial organization and legal liability has received much less attention compared to work on the interaction between liability and innovation or insurance (Viscusi and Moore, 1991a,b, 1993). The more recent work (Baumann and Heine, 2012) combines competition, innovation and liability for the case of risky products. Our approach is closer to Spulber (1989) which shows that the firm’s production level can influence the investment costs related to prevention. For example, investing in prevention depends on the cross effect of these investments and the monopoly’s production costs. Thus, potential injurers in very competitive markets may offer products that are insufficiently secure. Boyd (1994) shows that the optimal legal system is particularly sensitive to market structure and the characteristics of the firm’s risk reduction technology which result applies also to financial responsibility.

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¹ Each regime is defined later.
requirements. Our approach takes the opposite perspective. It shows that market equilibria depend on the nature of the enforced liability regimes.

The rest of the paper is organized as follows. In section 2 we introduce the model; in section three, we study the equilibrium outcomes under different liability regimes enforced by the regulator, and we compare their efficiency. Section 4 concludes the paper.

2. The Model

Consider an industry composed of two firms whose activities are is hazardous and may generate harm to human health and the environment. A rational regulator enforces a given civil liability regime in the form either of a strict liability or of a negligence rule; we define these regimes here, and describe them in detail in section three.

2.1 The producers’ production set and the accident costs

Each firm indexed \( i = 1,2 \) produces output \( q_i \). The production function \( \varphi(y_i) \), \( \varphi: [0,Y_i] \to \mathbb{R}_+ \), is assumed to be linear with identical marginal products \( a > 1 \):

\[
q_i = \varphi(y_i) = ay_i. \tag{1}
\]

A major accident may affect the firm’s activity in which case it induces damage proportional to production (see e.g. Dari-Mattiacci and De Geest, 2005). Damage per unit of output is \( D \); it is sufficiently large that \( a > 1 + 1/D \). Total damage for firm \( i \) is:

\[
d_i = Dq_i \tag{2}
\]

The probability of an accident is \( p(x_i) \) in firm \( i \) (with \( 1 - p(x_i) \) the probability of no-accident). Probability \( p(x_i) \) depends on the level of care \( x_i \) chosen by the firm. This is decreasing and convex in \( x_i \), or \( p'(x_i) < 0 \) and \( p''(x_i) \geq 0 \). Also, we assume \( p(0) \equiv 1 \) and \( p(Y_i) \equiv 0 \). Therefore, the expected cost of an accident \( EC(x_i) \) is \( 0 \cdot (1-p(x_i)) + d_i \cdot p(x_i)D\varphi(y_i) = p(x_i)Dq_i \).

Firms compete with production input \( y_i \) and prevention \( x_i \) as strategic variables under asymmetric capacity constraints \( Y_1 \neq Y_2 \):

\[
y_i + x_i \leq Y_i, i = 1,2. \tag{3}
\]

Considering the expression of prices \( \pi(.) = \) , for simplicity we assume an affine inverse demand function:

\[
\pi(\varphi(y_1) + \varphi(y_2)) = K - (\varphi(y_1) + \varphi(y_2)) = K - a(y_1 + y_2).
\]
The following inequalities ensure a non-negative market price and prevention levels between $0$ and $1$, $0 \leq K - a(Y_1 + Y_2) < D$, and $Y_i < 1 + 1/aD$.

Firm $i$’s profit function will depend on its involvement in the liability regime and the nature of that regime. Payoff $B_i(x_i, y_i; y_j): [0, Y_i] \times [0, Y_j] \to \mathbb{R}_+$ is stochastic:

$$B_i(x_i, y_i; y_j) = \varphi(y_i)\pi(\varphi(y_i) + \varphi(y_j)) - x_i - y_i - d_i. \quad (4)$$

It is such that:

$$B_i(x_i, y_i; y_j) = \begin{cases} 
\varphi(y_i)\pi(\varphi(y_i) + \varphi(y_j)) - x_i - y_i - D\varphi(y_i) & \text{(accident)} \\
\varphi(y_i)\pi(\varphi(y_i) + \varphi(y_j)) - x_i - y_i & \text{(no accident)}
\end{cases}. \quad (5)$$

Therefore, the expected profit is:

$$\mathbb{E}B_i(x_i, y_i; y_j) = \varphi(y_i)\pi(\varphi(y_i) + \varphi(y_j)) - x_i - y_i - p(x_i)D\varphi(y_i). \quad (6)$$

Firm $i$ maximizes the expected profit under the constraint (3) for non-negative inputs:

$$\max_{y_i \geq 0, x_i \geq 0} \mathbb{E}B_i(x_i, y_i; y_j), \quad (7)$$

s.t. $y_i + x_i \leq Y_i. \quad (7a)$

Without established tort law, firms do not forecast any care for repair purposes ($x_i \equiv 0$), and the model corresponds to a standard duopoly with a capacity constrained input as the strategic variable:

$$\max_{y_i \geq 0, x_i \geq 0} \varphi(y_i)\pi(\varphi(y_i) + \varphi(y_j)) - y_i, \quad (8)$$

s.t. $y_i \leq Y_i \quad (8a)$

2.2 Decision tree

Competition between potential injurers is modelled as a Cournot-Nash game with production inputs and prevention levels as the strategic variables. The game includes several stages:

**Stage-1**: The regulator enforces a specific civil liability rule on all firms.

**Stage-2**: Each firm maximizes its profits by choosing the optimal levels of prevention and production input, and therefore, of output given the other firm’s quantity and constraint on its available resources.
Stage-3: Given the liability regime, firms compare their equilibrium care level to the social level. Depending on the regime, they decide whether to reduce their equilibrium production level to comply with the social norm.

Stage-4: Nature comes into play, and a harm may occur.

Stage-4(a): No accident occurs, firms receive their pay-offs and the game stops.

Stage-4(b): An accident occurs and causes damage.

Stage-5: After Stage-4(b), the court makes its decision:

Stage-5(a): Under strict liability, the court looks for the existence of a causal link between the firm’s activity and the damage. If this link is positive, the judge determines the repair value which is assumed to be equal to the value of damage, and sentences the injurer to repair.

Stage-5(b): Under negligence, the court gauges the adequacy of the measures taken by the injurer relative to the social care level. If the injurer has complied with that level then it escapes liability, otherwise the judge will conclude that the injurer is liable, and must make reparations.

3. Equilibrium and Liability Regimes

A Cournot-Nash equilibrium is reached under the burden of a given legal regime as mentioned in the decision tree. To simplify the explanation, we begin with a strict liability regime which is used throughout the paper as the benchmark. Furthermore, note that, as the resource is limited, we cannot not expect compliance between the SCL and the optimal level of care that the firms choose (Shavell (1986), Beard (1990)).

3.1 Equilibrium and strict liability.

Strict liability consists of a burdening liability imposed on a party without identifying a fault. It is the opposite of negligence or tortious intent. The plaintiff needs only prove that the misdeed arose and that the injurer was liable. The regulator enforces strict liability for activities it considers to be innately hazardous. This should discourage

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6 However, note that the court may find it difficult to assess the adequacy of the level of prevention given the level of activity (Shavell, 1987; Shavell and Polinsky, 2005).
careless behavior and useless loss by inducing potential perpetrators to take all possible care. It also simplifies court decisions by lowering the necessity to find evidence.

Firms make their production and investment decisions simultaneously, and each knows the cost structure and the payoff function of its opponent. So, the game is one of complete but imperfect information, and competition is static (Tirole, 1988). To simplify the whole calculus, we use a linear probability of accident \( p(x_i) = 1 - x_i \) as in Hiriart and Martimort (2006), with \( 0 \leq x_i < 1 \). This assumption affects the minimum value of \( Y_i \) which must be greater than or equal to 1. Under strict liability, the program of firms \( i = 1, 2 \) are:

\[
\max_{y_i \geq 0, x_i \geq 0} \left[ K - a(y_1 + y_2) \right] a y_i - x_i - (1 - x_i) D a y_i
\]

s.t.
\[
y_i + x_i \leq Y_i, \tag{9a}
\]
\[
1 - x_i \geq 0. \tag{9b}
\]

**Lemma 1:** Under strict liability, the best responses for production input \( y_i^R \) and care \( x_i^R \) of firm \( i \) depend only on \( y_j^R \) (\( j \neq i, i = 1, 2 \)), and are such that \( y_i^R \) and \( y_j^R \) are strategic substitutes whereas \( x_i^R \) and \( x_j^R \) are strategic complements:

**Proof of Lemma 1:** The proof is given in appendix A. We obtain:

\[
y_i^R = \frac{Y_i}{2} + \frac{K - a(Y_i + y_j) - D}{2(a + D)} \quad \text{and} \quad x_i^R = \frac{Y_i}{2} + \frac{D - (K - a(Y_i + y_j))}{2(a + D)}, \tag{10}
\]

\( i = 1, 2, i \neq j. \)

From Lemma 1, if firm \( j \) exerts more market power by lowering its output, firm \( i \) maintains the price low by increasing its production input (\( \partial y_i^R / \partial y_j = -a / 2(a + D) \)) at the expense of safety: care decreases by the same amount (\( \partial x_i^R / \partial y_j = a / 2(a + D) \)); thus, the effect on the probability of accident in firm \( i \) is \( \partial p(x_i^R) / \partial y_j = -a / 2(a + D) \). This result which would occur e.g. if firm \( j \) were more constrained (e.g., \( Y_2 \) changes to \( Y_2' < Y_2 \)), shows the trade-off between market power and safety under strict liability. The Cournot-Nash solutions for production inputs \( (y_1^M, y_2^M) \) and equilibrium care levels are:

\[
y_1^M = \frac{2D(K+D(Y_1-1)+a(K+D(2Y_1-1-Y_2))}{(a+2D)(3a+2D)}, \tag{10}
\]
\[
y_2^M = \frac{2D(K+D(Y_2-1)+a(K+D(2Y_2-1-Y_1))}{(a+2D)(3a+2D)}. \tag{11}
\]
\[
x_i^M = \frac{(3a^2+6ad+2D^2)Y_i+aD}{(a+2D)(3a+2D)} + \frac{(D-K)}{(3a+2D)} \quad \text{for} \ i, j = 1, 2, i \neq j \tag{12}
\]

The Cournot-Nash equilibrium of the game is given by \( (q_1^M, q_2^M, x_1^M, x_2^M) \) where \( q_i^M = ay_i^M, i = 1, 2 \) and the market price:
\[\pi^M \equiv \pi(\varphi(y^M_1) + \varphi(y^M_2)) = K - a(y^M_1 + y^M_2) = \frac{K(a + D + 2aD + D(K - a(Y_1 + Y_2))}{3a + 2D}, \quad (13)\]

3.2 Negligence rule

An agent that fails to exercise due care, ethical rules, etc. commits negligence if this failure involves harm to one or several persons. An individual that suffers damage caused by another’s carelessness may be able to litigate in court to compensate for his or her losses (these losses include injury, physical or moral, harm to property and economic activities). However, if the potential injurer can provide evidence that it fulfilled its due care then it will escape any involvement in liability. Thus, the negligence rule requires a social standard of due care to be defined. Let us define its value before turning to the market equilibrium conditions.

3.2.1 The social cost of an accident and the socially first-best care level

The standard unilateral accident model proposes an endogenous determination of the socially optimal care level that results from maximization of the profit of a representative firm (a monopoly). This profit is

\[B(x) \equiv G - x - h(x), \quad (14)\]

where \(G\) is the firm’s constant global payoff; \(h(x)\) is the direct cost of an accident. Since \(G\) is constant, the program amounts to minimizing total cost \(x + h(x)\). The level of prevention \(x^0 > 0\) that minimizes the company’s total cost is such that:

\[h'(x^0) = -1. \quad (15)\]

In this approach, \(x^0\) is also the socially optimal level of prevention that minimizes the expected social cost of an accident \(ESC(x)\) which reduces to \(ESC(x) = x + h(x)\). The firm bears the costs of accident. Although important, this result relies on numerous restrictive assumptions: fixed gains, lack of competition, unlimited resources.

In our model, we can determine this level by considering a benevolent regulator that imposes a first best social level of care. We share the view that the regulator calculates an \textit{a priori} value for that level which firms should adopt. In the case of an accident, under negligence, the judges agree on that level and verify the adequacy of the injurer’s care. As under the unilateral standard accident model, the regulator seeks to minimize the expected social cost of an accident comprised of the prevention cost \(x\) and the expected cost of an accident \(p(x)H(x)\), where \(H(x)\) is the total damage which here depends on the level of care. Replacing \(y_i\) with \(Y_i - x_i\), the expected social cost becomes:
\[
ESC(x) = x_i + p(x_i)H(x_i) = x_i + (1 - x_i)D\alpha (Y_i - x_i), i = 1,2.
\] (16)

From the first order conditions, the socially optimal care level (SOCL) is
\[
x_i^0 = \frac{y_i}{2} + \frac{1}{2} - \frac{1}{2aD}.
\] (17)

In appendix B, we show that the internal solution concerning the level of care determined under strict liability does not fit with the socially first best level of care, i.e. \(x_i^0 > x_i^R\). This result means that competition leads the firms never to conform to the SOCL.

Then, the regulator knows that under a negligence rule, firms must increase their level of care to comply with the SOCL. It follows that the regulator exogenously may set a level of care lower than that level. There may be several reasons for this i.e. encouraging strategic or vital production, or avoiding too much involvement in civil reparation if the courts have previously agreed on an unattainable \(x_i^0\).

In what follows, the regulator chooses a minimum accident probability \(p^0\) as the standard of care. To comply with this firms should choose a level of care \(x^*, x^* \leq x^0\) such that \(\lim_{x \to x^*} p(x) = p^0\). We call \(x^*\), the Social Care Level (SCL). This is a similar situation described by Kolstad, Ulen and Johnson (1990) where the optimum level of an ex ante safety regulation should be less than the socially optimal level of precaution. Furthermore, we assume that the court knows the firms’ cost structures and will refer to this level to assess the injurer’s liabilities (i.e. there are no divergence between the judge and the regulator (Posner, 2003)).

### 3.2.2 Negligence rule and the market equilibrium

We consider as the baseline the previous equilibrium achieved under the strict liability regime \((q_1^M, q_2^M, x_1^M, x_2^M)\). We distinguish three cases according to whether the social norm \(x^*\) is lower or greater than \(x_i^M, i = 1,2\).

**Case 1:** \(x_i^M < x^* \forall i = 1,2\). The equilibrium levels of care that firms should chose under strict liability are less than the level the regulator requires under negligence. However, under negligence firms are supposed to avoid any liability; thus, they must increase safety by supplying a sufficient care level (at least \(x^*\)), and consequently using fewer production inputs. The firms’ payoff structure is denoted \(B\):

\[
B(x_i, y_i; y_j) = \begin{cases} 
(K - a(y_i + y_j))a y_i - x_i - y_i - p(x_i)Da y_j & \text{if } x_i < x^*, i = 1,2, i \neq j \\
(K - a(y_i + y_j))a y_i - x_i - y_i & \text{if } x_i \geq x^*, i = 1,2, i \neq j
\end{cases}
\] (18)
Payoff structure (18) resembles the standard negligence approach in economics. However, introducing strategic competition entails several changes, notably that the results are not in line with the regulator’s expectations, and a prisoner’s dilemma may exist. Indeed, when both firms comply with the SCL and produce respectively $q_i^1$ and $q_j^2$, then the total output $q_i^1 + q_j^2$ is lower than the output under strict liability:

$$\sum_{i=1}^{2}(q_i - q_i^1) + \sum_{i=1}^{2}(y_i - y_i^1) > 0 \tag{19}$$

However, a firm, for instance $i$, may exploit the decrease in total output strategically by raising its own supply within the interval $[q_i^1, q_i + (q_i^1 - q_j^1)]$ to increase its market power. Consequently, firm $i$’s expected payoff may increase. Then, firm $i$, which observes the SCL, may define its highest profits in $B(x_i, y_i; y_j^*)$. Therefore, firm $i$ will enhance its output by increasing its use of production inputs from $y_i^*$ to some $y_i^H > y_i^*$, and thus by decreasing its care level to $x_i^H < x_i^*$. More formally, $(x_i^H, y_i^H)$ is such that:

$$(x_i^H, y_i^H) \in \arg\max_{x_i^H > 0, y_i^H > 0} B(x_i, y_i; y_j^*)$$

Consequently, if $B^H(x_i^H, y_i^H; y_j^*)$ is greater than $B(x_i^*, y_i^*; y_j^*) = [K - a(y_i^* + y_j^*)] ay_i^* - x_i^* - y_i^*$, the payoff when both firms comply with the SCL, then it is not in $i$’s interest to conform to $x_i^*$. This is a prisoner’s dilemma which arises partly from the assumption of risk neutral firms. It is summarized in Table 1.

**Table 1:** The potential prisoner’s dilemma

<table>
<thead>
<tr>
<th>Firm i</th>
<th>Firm j</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C$</td>
</tr>
<tr>
<td></td>
<td>$B^*(x_i^1, y_i^1; y_j^1)$</td>
</tr>
<tr>
<td></td>
<td>$B^H(x_i^H, y_i^H; y_j^*)$</td>
</tr>
<tr>
<td>$C$</td>
<td>$B^*(x_j^1, y_j^1; y_i^1)$</td>
</tr>
<tr>
<td></td>
<td>$B^H(x_j^H, y_j^H; y_j^1)$</td>
</tr>
<tr>
<td>$\overline{C}$</td>
<td>$B^*(x_j^1, y_j^1; y_i^1)$</td>
</tr>
<tr>
<td></td>
<td>$B^H(x_j^H, y_j^H; y_j^*)$</td>
</tr>
<tr>
<td></td>
<td>$B^M(x_j^H, y_j^H; y_j^H)$</td>
</tr>
</tbody>
</table>

In Table 1, $C$ is “complies with the SCL”, and $\overline{C}$ is “does not comply”. E.g., in the bottom right cell $\overline{C}C$ none of the firms complies. There are four cases:

i) $CC$: both firms comply, and thus are free from any liability. Their payoffs are given above, $B^*(x_i^*, y_i^*; y_j^*) = [K - a(y_i^* + y_j^*)] ay_i^* - x_i^* - y_i^*$, $i = 1,2, i \neq j$. 

10
ii) \( C \bar{C} \) or \( \bar{C} C \): firm \( i \) (resp. \( j \)) does not comply and may extend its supply, given that firm \( j \) (resp. \( i \)) complies. We discuss the potential payoffs below.

iii) \( \bar{C} C \): none of the firms complies. The equilibrium payoff is denoted by \( B^H(x_i^H, y_i^H; y_j^H) \) which is less than \( B^*(x_i^*, y_i^*; y_j^*) \) and different from the payoff under strict liability \( B^M(x_i^M, y_i^M; y_j^M) \).

**Proposition 1.** Under negligence, when \( x_i^M < x^* \) \((i, j = 1, 2, i \neq j)\), two situations may arise:

a) If \( B^M(x_i^M, y_i^M; y_j^M) < B^*(x_i^*, y_i^*; y_j^*) \) and \( B^H(x_i^H, y_i^H; y_j^H) < B^*(x_i^*, y_i^*; y_j^*) \), where \( B^H(x_i^H, y_i^H; y_j^H) = \text{Sup}_{x_i,y_i}[B^h(x_i,y_i; y_j^H)] \) then:

i. The market equilibrium is \((q_i^h, q_j^h)\), \((i, j = 1, 2, i \neq j)\)

ii. The equilibrium price is, \( \pi^*, \pi^* > \pi^M \).

b) If \( B^M(x_i^M, y_i^M; y_j^M) < B^*(x_i^*, y_i^*; y_j^*) \) and \( B^H(x_i^H, y_i^H; y_j^H) > B^*(x_i^*, y_i^*; y_j^*) \), where \( B^H(x_i^H, y_i^H; y_j^H) = \text{Sup}[B^h(x_i,y_i; y_j^H)]; \) then, there is a prisoner dilemma:

i. The market equilibrium is \((q_i^H, q_j^H)\) with \( B^H(x_i^H, y_i^H; y_j^H) < B^*(x_i^*, y_i^*; y_j^*) \) and \( B^H(x_i^H, y_i^H; y_j^H) < B^M(x_i^M, y_i^M; y_j^M) \).

ii. The correspondent price \( \pi^H, \pi^H < \pi^* \) may be higher or lower than \( \pi^M \).

**Proof of proposition 1:** See appendix C

Part a of proposition 1 shows that firms derive no advantage from deviating from the SCL. Consequently, there is no prisoner’s dilemma. The negligence enforcement leads to a different equilibrium from that under strict liability. Moreover, the risk level is lower than under an objective liability regime, and the quantity also is lower because the price is higher. Part b of the proposition shows when a prisoner dilemma exists. Both agents are led to take the risk because they both ignore the fact that the other will deviate as in a classic prisoner’s dilemma (information is imperfect and complete). The equilibrium is the worst possible: the risk is higher than under negligence and outputs do not maximize the firms’ payoffs. Were the regulator to implement negligence, it would place the firms in a prisoner’s dilemma with a higher risk of accident than under strict liability.

**Case 2:** \( x_i^M > x^* \), \( i = 1, 2 \). Here, the equilibrium \((q_1^M, q_2^M, q_1^M, x_2^M)\) is such that \( x_i^M > x^* \Rightarrow y_i^M < y_i^* \), \( i = 1, 2 \), and the expected payoffs that correspond to negligence are
the following situation: Under negligence consequence of payoffs \( B \) would level by decreasing its safety level to converge Proposition regimes, assuming first that safety without any possibility proposition 1.

Proof is lower than the price corresponding to the Proposition which implies \( \pi^* < \pi^M \). The arguments are stated in the following proposition.

**Proposition 2.** Under negligence, when \( x_i^M > x^* \) (i = 1,2), the market equilibrium price is lower than the price corresponding to the SCL: \( \pi^* < \pi^M \).

**Proof of proposition 2:** the proof uses similar arguments to those stated in the proof of proposition 1.a.ii and 1.b.ii.

Under both strict liability and negligence, case 2 is such that firms overinvest in safety without any possibility to adjust care levels to the social value. Let us compare these regimes, assuming first that firms meet similar conditions under both regimes i.e., \((q_1^M, q_2^M)\) are identical. Without further demonstration, we can easily form the following proposition which establishes the convergence between both regimes.

**Proposition 3.** Under the conditions given in proposition 2, negligence and strict liability converge to the same equilibrium values.

**Proof of proposition 3:** the proof uses arguments similar to those set out above. Note that under negligence, neither firm 1 nor firm 2 has an interest in increasing its production level by decreasing its safety level to \( x_i^* \) because this would increase the price, and thus would contradict the Nash equilibrium of the game. Indeed, \( B(x_i^M, y_i^M, y_j^M) \geq B(x_i^k, y_i^k, y_j^k) \), \( \forall i = 1,2, k \neq M \). This last expression means that all other values than \((x_i^M, y_i^M)\), including the values drawn from the application of the SCL, lead to lower payoffs.

**Case 3:** \( x_i^M > x_i^* \) and \( x_j^M < x_j^* \), \( i,j = 1,2, i \neq j \). Firm \( i \) complies with the SCL whereas firm \( j \) does not; we must have \( y_i^M < y_i^* \) and \( y_j^M > y_j^* \). This case is a direct consequence of \( Y_i \neq Y_j \). Recall that under strict liability \( q_i^M = ay_i^M \) and \( q_i^M < q_i^* \), \( \forall i = 1,2 \). Under negligence however, \( x_j^M < x_j^* \) means that \( q_j^M > q_j^* \), and \( j \)'s payoff corresponds to the following situation:

\[
B(x_i, y_i; y_j) = \begin{cases} 
[K - a(y_i + y_j)]a y_i - x_i - y_j - p(x_j)Da y_j & \text{if } x_i < x_i^* \\
[K - a(y_j + y_i)]a y_j - x_j - y_i & \text{if } x_i \geq x_i^*
\end{cases} 
\]

(21)
a) If it turns out that \( B^*(\chi^M_j, \gamma^M_j; \lambda^M_i) < B^*(\chi^*_j, \gamma^*_j; \lambda^*_i) \), firm \( j \) chooses \( \chi^*_j \) to avoid any liability. This change exerts an influence on firm \( i \)'s action. Because now, its opponent chooses to supply \( \varphi(\gamma^*_j) \) rather than \( \varphi(\gamma^M_j) \), where \( \varphi(\gamma^*_j) < \varphi(\gamma^M_j) \). Firm \( i \) may see its probability of accident increase (following lemma 1) because as firm \( j \) decreases it production from \( \varphi(\gamma^M_j) \) to \( \varphi(\gamma^*_j) \) firm \( i \) may find it profitable to increase production. This production increase \( \varphi(\gamma^M_j) \) lies in the interval \( [\varphi(\gamma^*_j), \varphi(\gamma^M_j)] \) for \( B^*(\chi^M_j, \gamma^M_j; \lambda^M_j) \geq 0 \) with \( p(\chi^M_j) \leq p(\chi^*_j) \). The total quantity supplied at equilibrium is \( \varphi(\gamma^M_j) + \varphi(\gamma^*_j) \) and the equilibrium price \( \pi^M = K - (\varphi(\gamma^M_j) + \varphi(\gamma^*_j)) = 1 - a(\gamma^M_j + \gamma^*_j) \). Here, \( (\chi^M_j, \gamma^M_j) \) are the new Nash-equilibrium values associated to \( (\chi^*_j, \gamma^*_j) \). It then is obvious that as \( a(\gamma^M_j + \gamma^*_j) \leq a(\gamma^*_j + \gamma^*_j) \) then \( \pi^M \geq \pi^M \) because increasing care involves decreasing production input and therefore quantity; this may increase scarcity and lead to a rise in the equilibrium selling price.

b) Conversely, if \( B^*(\chi^M_j, \gamma^M_j; \lambda^M_i) > B^*(\chi^*_j, \gamma^*_j; \lambda^*_i) \), then \( j \) may consider that the probability of an accident is sufficiently low that it is in its interest is to choose \( (\chi^*_j, \gamma^*_j) \). This latter combination insures a higher production level as well as \( q^*_j > q^*_j \) with \( x^*_j < x^*_j \). The equilibrium price is the same as the price under strict liability, that is \( p^M \).

**Proposition 4.** If \( x^*_j > x^*_j \) whereas \( x^*_j < x^*_j \), \( i, j = 1, 2, i \neq j \), then firm \( i \) complies with the SCL because \( x^*_j > x^*_j \Rightarrow y^*_j < y^*_j \). Firm \( j \), however faces two situations according to whether \( B^*(\chi^*_j, \gamma^*_j; \lambda^*_j) < B^*(\chi^*_j, \gamma^*_j; \lambda^*_j) \) or \( B^*(\chi^*_j, \gamma^*_j; \lambda^*_j) > B^*(\chi^*_j, \gamma^*_j; \lambda^*_j) \).

a) When \( B^*(\chi^*_j, \gamma^*_j, \lambda^*_j) < B^*(\chi^*_j, \gamma^*_j, \lambda^*_j) \), it is in firm \( j \)'s interest to comply with the SCL and to choose inputs equal to \( (\chi^*_j, \gamma^*_j) \) such that \( \chi^*_j \geq x^*_j \). Consequently, the equilibrium will be \( (q^*_j, q^*_j) \) with for each firm, \( \chi^*_j \geq x^*_j \), \( i = 1, 2 \) and as \( y^*_j < y^*_j \), \( i = 1, 2 \), then \( (q^*_j, q^*_j) < (q^*_j, q^*_j) \) and for an equilibrium price, \( \pi^M > p^M \) such that \( \pi^M = K - (\varphi(\gamma^*_j) + \varphi(\gamma^*_j)) = K - a(\gamma^*_j + \gamma^*_j) \), with \( \pi^* \geq \pi^M > \pi^M \).

b) When \( B^*(\chi^*_j, \gamma^*_j, \lambda^*_j) > B^*(\chi^*_j, \gamma^*_j, \lambda^*_j) \), it is in firm \( j \)'s interest not to comply with the social care and to choose \( (\chi^*_j, \gamma^*_j) \) in which case the equilibrium price is \( \pi^M \).

**Proof of proposition 4:** the proof of b) is straightforward; a) deserves some explanation. Indeed, as the game is one of complete information, firm \( i \) knows that for firm \( j \), \( B^*(\chi^*_j, \gamma^*_j, \lambda^*_j) < B^*(\chi^*_j, \gamma^*_j, \lambda^*_j) \), and that firm \( j \)'s interest is to comply with the SCL. Then, the range of equilibrium values for prevention is \( \chi^*_j \geq x^*_j \). Firm \( i \) will determine the
corresponding equilibrium prevention level, $x^*_i$. It follows that the equilibrium is $(q_i^{M'}, q_j^{M'})$ such that $(q_i^{M'}, q_j^{M'}) < (q_i^{M}, q_j^{M})$. Indeed, firm $i$ may benefit from firm $j$’s limitation of supply, and may extend its production to $q_i^{M'}$ with $x_i^{M'} \geq x^*_i$ (i.e. respecting the SCL condition). We can then deduce the equilibrium price $\pi^M$, with $\pi^* \geq \pi^M > \pi^M$.

From the above proposition, it appears that by enforcing negligence the regulator affects the market equilibrium, unlike under strict liability where it is in firm $j$’s interest is to comply with the SCL. Proposition 4.a states that strict liability and negligence affect different equilibrium market values. Without any reference to the SLC, under strict liability each firm supplies the level that maximizes its profits; however, under negligence, the more exposed firm (here $j$) may be induced to reduce its supply which in turn, influences firm $i$ which may find it in its interest to increase its market share at the expense of safety. However, proposition 4.b shows that enforcement of negligence could have no impact on the exposed firm $j$; the latter may prefer not to comply with the SCL. Indeed, it prefers running the risk “as if” it was ruled by strict liability. In this case, negligence does not affect the market equilibrium, and this regime does not reinforce the safety level.

4. Conclusion

This paper analyzed the effects of different liability regimes on the strategic behavior of a Cournot duopoly involved in risky activities. Firms are capacity-constrained in that they use a limited input for both production and care. Introducing liability regimes in tort law induces changes in the firms’ behavior. Originally, the unilateral accident model showed that to avoid any involvement, firms must dedicate resources to increase their investment in prevention. In that framework, a representative firm’s objective is to maximize its care level and minimize its “primary” accident costs in the sense of Calabresi (1970) (the direct cost of an accident and the prevention level). Very few contributions study the effect of competition on liabilities except in the case of product liability.

We use strict liability as the benchmark under the rationale that in this regime the firms’ liability involvement is automatic in the case of an accident. Consequently, its interest is to minimize the accident probability by providing the highest level of care.
necessary to maximize its profit. Could the negligence rule lead to an increased level of care? By conforming to a supposedly known social care level, firms may hope to escape to their duty of repair in the case of an accident. However, because they are operating under complete but imperfect information, a prisoner’s dilemma could arise. The equilibrium of this game is detrimental to the market from the regulator’s perspective because firms will supply more output at the expense of the level of care. In this respect, negligence may not be a sufficient legal device to induce firms to take due care.

Then, enforcing liability regimes affects impacts on the level of exchange by encouraging or deterring firms’ supply. This affects the equilibrium price. If firms comply with the social care level under the negligence rule, then they will supply fewer goods and services than under strict liability. Indeed, the more stringent the safety standards, the higher the prices and the lower the market quantities.

This paper is a first attempt to study the relationships between liability regimes and competition beyond the issue of product liability. Several directions remain to be explored such as agents’ attitude to risk and uncertainty, the effect of different liability regimes when firms collude, and the oligopolistic dynamics when capacities are endogenous.

**Appendix A**

**Cournot-Nash solution under strict liability**

\[
\begin{align*}
\max_{y_i, x_i} & [K - a(y_1 + y_2)]a y_i - x_i - y_i - (1 - x_i)D a y_i, i = 1, 2 \\
\text{Under the constraints:} & \\
& y_i + x_i \leq Y_i \quad (A.2) \\
& x_i \leq 1 \quad (A.3) \\
& x_i, y_i \geq 0 \quad (A.4)
\end{align*}
\]

Let us consider the Lagrangian of this program for firm 1:

\[
\mathcal{L}(x_1, y_1, \lambda_1, \lambda_2) = [K - a(y_1 + y_2)]a y_1 - x_1 - y_1 - (1 - x_1)D a y_1 + \lambda_1 (Y_1 - y_1 - x_1) + \lambda_2 (1 - x_1),
\]

The Karush-Kuhn-Tucker conditions are:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial x_1} & \leq 0 \Rightarrow -1 + aD y_1 - \lambda_1 - \lambda_2 \leq 0 \quad (A.6) \\
\frac{\partial \mathcal{L}}{\partial y_1} & \leq 0 \Rightarrow aK - 2 a^2 y_1 - a^2 y_2 - 1 - (1 - x_1)D a - \lambda_1 \leq 0 \quad (A.7)
\end{align*}
\]
\[ x_1 \frac{\partial L}{\partial x_1} = 0 \Rightarrow x_1(-1 + aD_y - \lambda_1 - \lambda_2) = 0 \quad (A.8) \]
\[ y_1 \frac{\partial L}{\partial y_1} = 0 \Rightarrow y_1(aK - 2a^2y_1 - a^2y_2 - 1 - (1-x_1)Da - \lambda_1) = 0 \quad (A.9) \]
\[ x_1 \geq 0 \quad \text{(A.10)} \]
\[ y_1 \geq 0 \quad \text{(A.11)} \]
\[ \frac{\partial L}{\partial \lambda_1} \geq 0 \Rightarrow Y_1 - y_1 - x_1 \geq 0 \quad \text{(A.12)} \]
\[ \frac{\partial L}{\partial \lambda_2} \geq 0 \Rightarrow 1 - x_1 \geq 0 \quad \text{(A.13)} \]
\[ \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow \lambda_1(Y_1 - y_1 - x_1) = 0 \quad \text{(A.14)} \]
\[ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow \lambda_2(1 - x_1) = 0 \quad \text{(A.15)} \]
\[ \lambda_1 \geq 0 \quad \text{(A.16)} \]
\[ \lambda_2 \geq 0. \quad \text{(A.17)} \]

We seek an interior solution for \( x < 1 \) under the capacity constraint; thus, suppose that \( \lambda_1 > 0 \) \( (\Rightarrow y_1 = Y_1 - x_1 \text{ in A.14}) \) and A.13 is not binding \( (\Rightarrow \lambda_2 = 0) \). Therefore, \( \lambda_1 = aDy_1 - 1 \) in A.8. We first discard \( (x_1, x_2) = (0, 0) \) \( (y_1 = Y_1, y_2 = Y_2) \) because A.9 would be equal to \( aY_1(K - a(2Y_1 + Y_2) - D) = aY_1(K - a(Y_1 + Y_2) - D - (a + D)Y_1) \), which is negative as \( K - a(Y_1 + Y_2) - D < 0 \) by assumption. But A.9 must be equal to zero. Therefore \( (x_1, x_2) = (0, 0) \) is impossible. Note also that \( y_1 \), and therefore \( y_2 \), cannot be equal to zero, otherwise \( \lambda_1 \) would be negative which would contradict condition A.16. We thus solve the following system:

\[-1 + aD_y - \lambda_1 = 0. \]
\[ aK - 2a^2y_1 - a^2y_2 - 1 - (1-x_1)Da - \lambda_1 = 0. \]
\[ Y_1 - y_1 - x_1 = 0. \]

where \( 0 < x_1 < 1 \). The best response functions for \( y_1^R \) and \( x_1^R \) are:

\[ y_1^R = \frac{y_1}{2} + \frac{K - a(Y_1 + y_2) - D}{2(a+D)} \text{ and } x_1^R = \frac{y_1}{2} + \frac{D - (K - a(Y_1 + y_2))}{2(a+D)}, \]

and for \( y_2^R \) and \( x_2^R \):

\[ y_2^R = \frac{y_2}{2} + \frac{K - a(Y_2 + y_1) - D}{2(a+D)}, \text{ and } x_2^R = \frac{y_2}{2} + \frac{D - (K - a(Y_2 + y_1))}{2(a+D)}. \]

Appendix B
Demonstration of the inequality between the internal level of care and the socially first best care level.

We adopt the well-known view that the regulator a priori calculates the value for the socially first best level of care that firms should choose. Accordingly, considering the unilateral standard accident model, the regulator looks to minimize the social cost of an accident which comprises the prevention cost $x$ and the expected cost of an accident $p(x)h(x)$, where $h(x)$ is the total damage which here depends on the level of care.

Using our notations and replacing $y_i$ by $Y_i - x_i$, the social cost can be written as

$$SC(x) = x + p(x)H(x) = x_i + (1 - x_i)D \cdot (Y_i - x_i) \ i = 1, 2.$$  

Then, from the first order conditions we can deduce the socially first best of care $x^*_i$:

$$x^*_i = \frac{Y_i}{2} + \frac{1}{2} - \frac{1}{2aD}$$  \hspace{1cm} (B.1)

The conditions for $1 > x^*_i \geq 0$ are satisfied. Parameters $a$ and $D$ are greater than 1 by assumption, therefore $1/2 - 1/2aD > 0$ and thus $x^*_i > 0$. Moreover, $Y_i < 1 + 1/aD$ by assumption, so $x^*_i < 1$. We can show that this optimal level $x^*_i$ is greater than $x^R_i$ for firm $i$, i.e. $x^0_i > x^R_i$, where $x^R_i = \frac{Y_i}{2} + \frac{D -(K - a(Y_i + y_j))}{2(a + D)}$ (see appendix A). Subtracting $x^R_i$ from $x^*_i$ we obtain

$$x^0_i - x^R_i = \frac{1}{2} - \frac{1}{2aD} - \frac{D -(K - a(Y_i + y_j))}{2(a + D)}.$$  

Note that $D - (K - a(Y_i + y_j)) < D - (K - a(Y_i + Y_j)) < D$, $\forall i, j$, since $K - a(Y_i + Y_j) > 0$ by assumption. Therefore,

$$x^0_i - x^R_i > \frac{1}{2} - \frac{1}{2aD} - \frac{D}{2(a + D)}.$$  

Reducing the right-hand side of this inequality to a common denominator then factoring out $1/2$ we obtain

$$\frac{1}{2} - \frac{1}{2aD} - \frac{D}{2(a + D)} = \frac{1}{2} \left(\frac{a^2D - a - D}{aD(a + D)}\right).$$  

But $a$ and $D$ are both greater than 1 with $a$ sufficiently large relative to $1/D$ ($a > 1 + 1/D$ by assumption); thus, $a^2D > aD + a$. Therefore, $a^2D - a - D > aD + a - a - D = D(a - 1) > 0$, and consequently, $x^0_i - x^R_i > 0$.

Appendix C

Proof of proposition 1
Part 1.a.i: to avoid any liability, firms \( i \) and \( j \) both decide to comply with the SCL and enforce \((x_i^i, y_i^i)\), where \( x_i^i > x_i^M \) and \( y_i^M > y_i^i \). Consequently, \( B^*(x_i^i, y_i^i; y_j^j) > B^M(x_i^M, y_i^M; y_j^M) \) where \( B^M(x_i^M, y_i^M; y_j^M) = [K - a(y_i^M + y_j^M)]a y_i^M - x_i^M - y_i^M - p(x_i^M)D a y_i^M \) is net of the accident cost; with compliance firms avoid this cost.

Part 1.a.ii: as the equilibrium quantities are \( q_i^* = a y_i^* < q_i^M \) \((i = 1, 2)\) then \( q_1^* + q_2^* < q_1^M + q_2^M \) (from the definition of the inverse demand function). It follows that \( \pi^* > \pi^M \). Equilibrium \( (q_i^*, q_j^*) \) is a Nash equilibrium and there is no prisoner’s dilemma.

Part 1.b.i: \( B^M(x_i^M, y_i^M, y_j^M) < B^*(x_i^*, y_i^*, y_j^*) \) but \( B^H(x_i^H, y_i^H, y_j^j) > B^*(x_i^*, y_i^*, y_j^*) \).

Firms do not communicate. Neither has an interest in complying with the SCL. Since each firm ignores what its rival will do, it chooses not to comply and to supply \( \varphi(y_i^H), i = 1, 2 \). Consequently, total output is \( \varphi(y_i^H) + \varphi(y_j^H) = q_i^H + q_j^H \). However, since \( q_i^H > q_i^M, i = 1, 2 \), firms risk being liable; thus \( B^H(x_i^H, y_i^H, y_j^j) < B^*(x_i^*, y_i^*, y_j^*) \), \( i, j = 1, 2, i \neq j \). It is optimal to deviate from the SCL only if both firms choose the strict liability solution \((q_i^H, q_j^H, p^H)\) which can be achieved by putting in place \((x_i^M, y_i^M)\), \( i = 1, 2 \). Other solutions are dominated, then \( B^M(x_i^M, y_i^M, y_j^M) > B^H(x_i^H, y_i^H, y_j^H) \).

Part 1.b.ii: in the case \( \pi^H < \pi^* \), the inequality is immediate, given \( q_i^H > q_i^M, i = 1, 2 \). If \( \pi^H \) is higher or lower than \( \pi^M \), then if \( q_i^H > q_i^M, i = 1, 2, \pi^H < \pi^M \); otherwise \( \pi^H \geq \pi^M \).
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