# CIVIL LIABILITY, KNIGHT'S UNCERTAINTY AND NON-DICTATORIAL REGULATOR

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# CIVIL LIABILITY, KNIGHT'S UNCERTAINTY AND NON-DICTATORIAL REGULATOR

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**Abstract:** *This paper reviews the foundations of the unilateral standard accident model under Knightian uncertainty. It extends the Teitelbaum (2007)'s seminal article (who introduces radical uncertainty) by expanding it from producers to victims and from the probability distribution of accidents to the scale of damage. Mainly, it also considers a regulator who aggregates the agents' preferences (Neghisi (1960) type). Under the condition that the troublemakers' resources are sufficient to cover the damage, the article shows that uncertainty does not preclude, first, the determination of a socially optimal level of care, and second, whatever the civil liability regime (strict liability or negligence) it shows that they determine the same level of socially first-best care. The solution is inefficient only when the polluter's wealth is insufficient to repair the victim's losses*.

**JEL codes**: D62, K13, K23, K32, Q52, Q58.

**Keywords:** unilateral accident, tort law, safety, large risks, ambiguity, pessimism and optimism, strict liability, negligence, ultra-hazardous activities.

## Introduction

Initiated by Ronald Coase (1960) and developed by Calabresi (1970), Brown (1973) and especially Shavell (1980), (1982), (1987b), the standard unilateral accident model constitutes the backbone of the economic analysis of tort law. Initially, created by lawyers and judges, tort law legally forces the wrongdoers to compensate their victims' losses. Beyond compensation, the economists showed that the tort law challenge is to understand how the prospect of paying heavy repairs to victims can motivate potential tortfeasors to provide the highest prevention level against risk. Hence, the model is simple: a potential wrongdoer performs activities likely to cause harm to other agents (victims). The prospect of losses due to compensation must encourage him (her) to take preventive measures to enhance his business from hazards.

Basically, the present paper reviews the foundations of the unilateral accident model under Knightian uncertainty or, still, "ambiguity". Indeed, in recent years, various contributions have extended the unilateral model of accident to radical uncertainty[1]. The Teitelbaum (2007)'s pioneering representation formalizes Knightian uncertainty applied to tort law. He substitutes to the classical Savage Expectation Utility, the new developments of ambiguity theory. In this model, uncertainty leads to ambiguous choices as in the Ellsberg's paradox .To formalize ambiguity, Teitelbaum (2007) assumes that the polluters' utility function takes the form of a so-called "neo-additive capacity" that allocates specific weights to extreme earnings (maximum and minimum payoff), and the expected gain (expectation). He shows that both a strict liability and a fault-based regime do not achieve a socially optimal prevention level. However, negligence seems more efficient than strict liability. Other contributions in this vein confirm the relationship between uncertainty and care level inefficiency. For instance, Franzoni (2012) considers the case of ambiguous risk where ambiguity is graded because of the existence of alternative distributions on the accidents likelihood. He analyses unilateral and bilateral accident models. He shows that, under strict liability, damage increases with rising ambiguity. In contrast, under negligence, safety standards increase but only when the injurer's perceived ambiguity reduces. Furthermore, when the polluter feels both a lower degree of risk and ambiguity aversion than the victim and a lower estimate of the damage likelihood, then, strict liability is more efficient than negligence.

[1] "Unilateral" means that the victims cannot buffer themselves from the accidents risk. Consequently, this relieves them from any liability, unlike the bilateral accident model.

In a somewhat different model, Langlais (2012) also shows that Knight's uncertainty leads to socially inefficient level of care. He considers a global non-insurable risk where the polluters invest in reducing risk technologies. Compared to victims, the polluter feels a fewer degree of aversion to risk and ambiguity. Then, his estimate of the prejudice likelihood corresponds also to a lower ambiguity degree. Langlais' model is based on supposed pessimistic and risk-averse agents. Agents are maximizers Rank Dependent Expected Utility. He shows that the required security level is higher than in a neutral to risk economy and that no liability regime is significantly efficient.

Similarly to the above approaches, our paper also introduces radical uncertainty in the unilateral accident model. However, it clearly differs from them because it shows, first, that uncertainty does not prevent the formation of a socially optimal care level and, second, that strict liability and negligence are equivalent when the polluter's wealth covers damage. Hence, inefficiencies appear when the damage costs are higher than the tortfeasor's wealth. This issue comes from the nature of the regulator's utility function. The latter is benevolent, omniscient but of Negishi (1960)'s type. This means that the social utility function aggregates the agents' preference conversely to the usual models that consider a neutral to risk regulator.

Furthermore, technically, compared to Teitelbaum (2007) and other mentioned works, our model extends the field of uncertainty. Thus, uncertainty integrates the accident probabilities (Teitelbaum (2007)), but also the question of the scale of damage which , here, is comprised in an interval and cannot be represented by only a mathematic expectation as in the usual case. Furthermore, ambiguous feelings concerns both the polluters (as in Teitelbaum (2007), Langlais (2012), Franzoni (2012)), and the victims.

However, this model does not examine in depth the issue of equivalence of a strict liability regime versus negligence by considering the distortions of appreciation between regulator and court, for example, or errors made by the judges. Pursuing such a discussion on this topic would led us too far (see Sommer (1983), Shavell (1980 b), Polinsky (1980), etc.).

## 1. Extension of the uncertainty area

In the present model, the uncertainty area is twofold. First, as in Teitelbaum (2007)'s article, radical uncertainty applies to beliefs associated with the probability distribution of accidents (see also Franzoni (2012) and Langlais (2012)). Such uncertainty implies that this probability distribution is common knowledge between the regulator and the injurers. However, the latter may cast doubts regarding the effective probability of accident. These doubts depend of the injurer's level of optimism and the confidence in the regulator's assessment. This discrepancy raises the question of how assessing accident risk in hazardous industries. For example, when the industrial operator controls the risky technology, the question of how transferring this knowledge to the regulator is at stake. Teitelbaum's uncertainty reveals this asymmetry: the government specialized offices define a specific accident probability considering the given hazardous activity. However, the operators or producers that manage it may assess differently its risk. In our model, this uncertainty field extends to potential victims. Generally, in the unilateral accident models, the victims do not act directly on the level of protection: they "disappear" from the decision process. Here, excluding the victims' preferences is no longer possible because the regulator aggregates the individuals' utility to define the social utility function. Hence, their beliefs influence the first-best care level.

The second uncertainty source comes from the scale of damage. Indeed, generally, experience lacks to assess the effective extent of technological collapses. Disaster data is often insufficient because each case is specific. For example, it is difficult to assess the negative consequences of accidently (or structurally in the case of pollution associated to production) spreading toxic effluents in water, soil, air, etc. Hence, the costs of these disasters are difficult to predict. Economic theory considers either that these are given average data (as an expected value), or that this one varies according the activity or protection level[2]. However, each point of the deterministic value is calculated from an implicit given probability distribution. Therefore, this distribution can be questioned as it may seem plausible or not. Indeed, assessing the costs of damage after a catastrophe always cause problems. Hence, for Shavell in "Economic Analysis of Accident Law", « *Once it has been established that an injured is liable, the amount he is to pay the victim must be determined*" Shavell (1987, p. 127). Shavell emphasizes the importance of losses evaluations by courts. In addition, the harm to individuals are always difficult to assess (Rogers, Bichaka and Balch (1991))) and the damage depends on the nature of the destroyed goods (Kopp and Smith (1993)). More precisely, if the reference to a market can help estimating private goods or property losses, this is hardly the case for semi-public and public goods. Therefore, giving an assessment of the value of damage before the accident is a hard task. Consequently, forming divergent estimates or beliefs about the damage scale is legitimate. Hedonic methods can be used, but the intensity of the sinister also affects the value of goods to be estimated (Maes (2010)). Furthermore, Shavell stresses that courts face with huge uncertainty when they have

[2] See for instance Dari-Mattiacci and Parisi, (2003).

to determine the financial losses: « *By contrast, because non pecuniary losses cannot be observed directly, they are difficult for courts to estimate.* » (Shavell, (1987) op. cit. p. 134). Consequently, if assessing ex-post damage is a complex task, the challenge is much higher when, ex-ante, the stakeholders must estimate it. Then, it is reasonable to consider that polluters and victims assess the costs of a major damage inside an interval and form beliefs once given the "official" data. In the present context, this forms the ambiguity theory basis.

We develop here the beliefs about major damages. Let $\mathcal{E}$ be the finite set of the states of nature that corresponds to a maximum damage involved by a major accident. $\mathcal{E}$ is included in the $\sigma$-algebre of $\mathcal{E}$. We define a set of $A$ issues (value of damages) and a set of simple functions $\Phi$ that verify the following point-to point mapping: $\Phi = \{f: \mathcal{E} \rightarrow A\}$. These ones map the damage set in $A, A \subset \mathbb{R}$, such that for each element $a_v \in A$, ($a_v$ is also called an act), we define the following ordering between the acts: $a_1 \geq a_2 \ldots \geq a_n$. Then, if $E_p(a)$ is the expected value of damage, now, the damage function writes as:

(1) $$E_p(a) = \int_{\overline{d}}^{l} a\, p(a)da$$

(where $l$ and $\overline{d}$ are defined below).

And the neo-additive capacity is (see appendix 1 for details):

(2) $$\mu(\,A\,/\,p, \delta, \alpha) = \begin{cases} 0 \;\; for \;\; A = \emptyset \\ \delta\alpha\nu_0(A) + \delta(1-\alpha)\nu_1(A) + (1-\delta)p(A) \; for \; \emptyset \subsetneq A \subsetneq \mathcal{E} \\ 1 \; for \; A = \mathcal{E} \end{cases}$$

for $\alpha, \delta \in [0,1]$.

Let $\nu_0(A) = \mathrm{Inf}(f) = \overline{d}$ be the lowest damage cost and $\nu_1(A) = \mathrm{Sup}(f) = l$), the highest one. The values $\delta$ and $\alpha$ represent the weight that the injurer allocates to the extreme events where $\delta$ is the preference for ambiguity while $\alpha$ stands for the optimism degree[3]. Then for $\emptyset \subsetneq A \subsetneq \mathcal{E}$ the neo-additive capacity is:

(3) $$\mu(\,.\,) = \delta\alpha\overline{d} + \delta(1-\alpha)l + (1-\delta)p(A)$$

We obtain a Choquet's integral by integrating the capacity $\mu(\,.\,)$ that represents the expected costs related to a major accident:

(4) $$V_p(A/p, \delta, \alpha) = \delta\alpha\overline{d} + \delta(1-\alpha)l + (1-\delta)E_p(a)$$

[3] See Teitelbaum(2007) for a more precise explanation about this point.

Hence, the Choquet integral of a neo-additive capacity consists of the following elements, i) The maximum value of the costs associated to a major accident( $l$), ii) Their minimum ($\overline{d}$),iii) Their expected value $\left(E_p(a)\right)$.

Optimism and pessimism are associated with the accident scale. Optimism implies a high value of α, because it is associated to the lowest damage ($\overline{d}$) occurrence, while pessimism $(1-\alpha)$ is linked to the highest damage cost $l$. Damage spans the entire $A$ spectrum. For instance, when $\alpha = 0$, the injurer is fully pessimistic, then:

$$V(\,A/p,\delta,0) = \delta l + (1-\delta)E_p(a)$$

Then, this expression only depends on his attitude to ambiguity $\delta$. When $\delta = 0$, (full aversion for ambiguity) then the capacity comes to $E_p(a)$ and the probability distribution becomes: $\mu(A\,/\,p, 0, \alpha) = p(A)$. Conversely, the higher $\delta$, the lesser the tortfeasor will be confident in the expected value of the damage costs, $\delta = 1$ means a complete distrust in it:

(5) $$V_p(A\,/\,p, 1, \alpha) = \alpha\overline{d} + (1-\alpha)l$$

This expression is also called the Hurwitz criterion, it is weighted by the injurer's level of optimism and pessimism.

Before beginning the study, let us have a look on the standard accident mode. Here, the omniscient, benevolent, neutral to risk regulator has to minimize the total accident costs: $x + p(x)E_p(a)$ or $x + p(x)\bar{l}$, (where $E_p(a) = \bar{l}$, in the following). Hence, $x^*$ the socially optimal level of care solves :

$$\min_{x>0} x + p(x)\bar{l}$$

Then, assuming that $x^*$ is positive, this value is drawn from the first-order condition:

$$-p'(x)\bar{l} = 1$$

This last equation requires that the marginal reduction in expected accident losses equal the marginal cost. We can easily see that, under the standard case, the regulator do not care about the preferences of the polluters and victims. Furthermore, the level of care is independent from the liability regime

## 2. The Model

As Shavell (1985), we assume that all polluters are identical and this also the case for the victims. This assumption allows connecting generically a representative polluter and a representative victim. The first ones sell to the second ones a product whose manufacturing process poses serious risks to human health, damage to property and to the environment. These risks affect the victims' utility functions. In our model, the regulator maximizes a social utility function of Negishi (1960)'s type that builds by aggregating individual preferences. Aggregation is simplified because, as the agents are identical, their utility marginal value is identically weighted. The regulator's role is then to induce the agents to reveal their preferences and to determine the socially first-best level of care by maximizing the aggregated social utility function.

The government's duty is to enforce the best civil liability regime, i.e. the one that must prompt the potential tortfeasors at applying the highest level of care at the lowest accident costs. To deal with this question, the basic model assumes that a dictatorial, omniscient, benevolent and neutral to risk regulator determines the socially optimal prevention level to which polluters must comply. When the regulator's first-best prevention level corresponds to the one naturally chosen by the tortfeasor, the solution reaches the socially first best level of care. This involves that, both, polluter and regulator are risk neutral. However, when the troublemaker is averse to risk, his solution does not match with the regulator's one and the equilibrium is no longer socially first-best (Shavell (1982)).

Consequently, contrary to the standard model, the regulator is not a dictator. This difference has important implications. Here, the regulator aggregates the agents' utilities to determine the social first-best. Other approaches than the Negishi's one are also based on the aggregation of preferences to represent the social utility function. Harsanyi (1955) has inspired a revival in this direction (Gilboa, Samet, and Schmeidler (2004), Fleurbaey and Mongin (2012), Danan, Gajdos and Tallon (2014)). Here, the aggregation is even easier than the agents in their own category (polluters and victims) are identical.

The hypothesis of identical agents is not specific to our representation. If that were so, it would weaken the model. In fact, a similar assumption is found in Shavell (1982) and is common to the entire civil liability stream. Conversely, when the dictatorial regulator owns a specific utility function (Shavell (1982)) that does not represent the agents' preference, the only position that allows reaching an efficient first-best level of care is when the injurer is also risk neutral.

### 3.1 Notations and assomptions

- $x$ represents the level of care (or prevention), it is a cost corresponding to an effort.
- $\Psi(x)$ is the injurer's Expected Choquet Utility (ECU) for an effort corresponding to $x$.
- $u$ is the injurer's wealth, $u > 0$,
- $\Phi(x)$ is the victim's ECU for a care effort corresponding to $x$.
- $v$ is the victim's wealth, $v > 0$,
- $p(x)$ is the probability of an accident for a care level $x$, with $p'(x) < 0$ and $p''(x) > 0$.
- $d$ cost of damage due to an accident caused by the polluter and $l$ is the level of repair that he can effectively make, with $d > 0, u > l, d \geq l$.
- $d, l$ notations that express the costs of a damage linked to an accident caused by the injurer, $d, l > 0, u > l, l \geq d$.

- Liability regimes indexes:
  - $NLI$ : No-liability,
  - $SL$ : Strict liability,
  - $SLC$ : Ceiled or capped strict liability,
  - $NR$ : Negligence rule,
  - $s = \{NR, NLI, SL, SLC\}$, (each regime specificity will be stated when studying each situation).
- Construction of the expected Choquet utilities of polluters and victims:
  These relate to beliefs about the damage and the accident distributions. The following table allows synthesizing the agents' beliefs.

| | Beliefs about the extent of major damage | | Belief on the probability of accidents distribution | |
|---|---|---|---|---|
| Degrees | {Optimism, Pessimism} | {Preference, ambiguity aversion } | {Optimism, Pessimism} | { Preference, ambiguity aversion } |
| Injurer | $\{\alpha, 1-\alpha\}$ | $\{\gamma, 1-\gamma\}$ | $\{\beta, 1-\beta\}$ | $\{\theta, 1-\theta\}$ |
| Victim | $\{\varepsilon, 1-\varepsilon\}$ | $\{\eta, 1-\eta\}$ | $\{\sigma, 1-\sigma\}$ | $\{\omega, 1-\omega\}$ |
| | | | | |

(where $\epsilon, \tau \in (0,1), \epsilon = \{\alpha, \beta, \varepsilon, \sigma \}, and\ \tau = \{\gamma, \theta, \eta, \sigma\}$)

- The injurer :

Uncertainty that injurers face with is twofold. The first one concerns the damage scale. This interval is independent from the choice of the care level (as when damage is an expected value in the standard model). However, this assessment depends on the current liability regime. Indeed, according the current liability, the polluter may be either fully exempted or partially free from liability, or still fully involved. In a generic way, this situation is described by $\Theta_P^s$, $s = \{NR, NLI, SL, SLC\}$, the ECU associated to the major damage. Then, from (3):

$$(6) \qquad \Theta_P^s = V_p(A/p, \delta, \alpha) = \begin{cases} \alpha\gamma\overline{d} + (1-\alpha)\gamma l + (1-\gamma)\overline{l} \text{ for } s = SL \ (a) \\ c \text{ for } s = SLC \quad (b) \\ 0 \text{ for } s = NLI \ (c) \\ 0 \ or \ \alpha\gamma\overline{d} + (1-\alpha)\gamma l + (1-\gamma)\overline{l} \ \text{ for } s = NR \ (d) \end{cases},$$

Under strict liability (SL), the expectation of repairs for the polluter is $(6)(a)$, when the liability is capped (SLC), the amount is $6(b)$. It is null for a full exemption of the injurers' liability $6(c)$ and, for negligence, the amount is either 0 or more as shows it $6(d)$.

The second source of uncertainty concerns the probability distribution of accident. From this point of view, the processing is entirely consistent with that of Teitelbaum (2007). Thus, considering that the value of the payoff function without accident is: $\text{Sup}\{\text{ f}\} = (u - x)$, and $Inf\{\,f\} = (u - x - \Theta_P^s\,)$ is the one reduced by repairs.

(7) $\Psi^s(x) = \beta\ \theta Sup\{\,f\} + (1-\beta)\theta\ Inf\{\,f\} + (1-\theta)\{(1-p(x)\{Sup\ f\} + p(x)\{Inf\ f\})\}$, for, $\text{s} = \{\text{NR, NLI, SL, SLC}\}$.

Replacing the maximum and minimum values, we get:

(8) $\Psi^s(x) = \beta\ \theta(u-x) + (1-\beta)\theta(u - x - \Theta_P^s\,) + (1-\theta)\big\{\big(1-p(x)\big)(u - x) + p(x)(u - x - \Theta_P^s)\big\} = u - x - \beta(1-\theta)\Theta_P^s - (1-\theta)\Theta_P^s p(x)$.

After simplification, the injurer's program consists in defining a care level that will minimize the accident probability and maintaining the lowest prevention cost:

(9) $\underset{x\geq 0}{Max}\{u - x - (1-\beta)(1-\theta)\Theta_P^s - (1-\theta)\Theta_P^s p(x)\ \}$

As the wealth $u$ is given, this amounts to minimizing:

(10) $\underset{x\geq 0}{Min}\{x + (1-\beta)(1-\ \theta)\ \Theta_P^s + (1-\ \theta)p(x)\Theta_P^s\}$,

for $\text{s} = \{\text{NR, NLI, SL, SLC}\}$

- The victim(s)

As the regulator aggregates the agents' preferences, it makes sense to include the victims' perception of damage. Indeed, these estimates influence the first-best level of care and the current liability regime influences the level of repairs by the compensation that the victims receive. Thus, the victims' ECU becomes:

(11) $\Theta_V^s = V_\phi = \varepsilon\eta\overline{d} + (1-\varepsilon)\eta l + (1-\eta)\overline{l}$, for s = {NR, NLI, SL, SLC}.

$$\Theta_P^s = V_p(A/p, \delta, \alpha) = \begin{cases} 0 \text{ for } s = SL \ (a) \\ \varepsilon\eta\overline{d} + (1-\varepsilon)\eta l + (1-\eta)\overline{l} - c \text{ for } s = SLC \ (b) \\ \varepsilon\eta\overline{d} + (1-\varepsilon)\eta l + (1-\eta)\overline{l} \text{ for } s = NLI \ (c) \\ 0 \text{ or } \varepsilon\eta\overline{d} + (1-\varepsilon)\eta l + (1-\eta)\overline{l} \text{ for } s = NR(d) \end{cases}$$

The (11) (a), (b), (c), (d) cases correspond to the above (6) cases and a further explanation is not necessary.

This one is integrated in their ECU :

(12) $\phi^s(x) = \{\sigma\,\omega\,v + (1-\sigma)\omega(v - \Theta_V^s) + (1-\omega)\{p(x)(v - \Theta_V^s) + (1 - p(x))(v)\}\} =$

$$= v - (1-\omega)\sigma\Theta_V^s - (1-\omega)\Theta_V^s p(x), \text{ for } s = \{NR, NLI, SL, SLC\}$$

- The regulator

The regulator does not impose his preferences on the basis of a priori given utility function. This means that he determines the socially first-best level of care after the aggregation of the agent's utility functions. As by assumption, all producers are identical as are identical the victims, the regulator simply aggregates the utilities of representative agents. Let $EWS^s(x)$ be the social utility function, then, the regulator's program is to maximize this function:

(13) $EWS^s(x) = \underset{x\geq 0}{Max}\{\Psi^s(x) + \phi^s(x)\}$ for $s = \{NR, NLI, SL, SLC\}$

For now, we cannot further specify the social utility function because this one depends on the current civil liability regime.

### 3.2 The first-best solutions to the unilateral accident question

"Solutions" in the plural means that accident costs are not independent from the liability regime, unlike the standard model. It is the point that we study by now. As the regulator is benevolent, he determines the Pareto-efficient care efforts.

Considering that $u_a$ and $v_a$ correspond respectively to the wealth of injurer and victims without accident occurrence and $u_n$ and $v_n$ to their wealth after an accident, the regulator's program becomes:

$$\max_{x, v_a, v_n, u_a, u_n,} [(\sigma\omega + (1-\omega)(1-p(x)))\phi^s(v_a) + ((1-\sigma)\omega + (1-\omega)(p(x)))\phi^s(v_n)] \quad (14 \text{ a})$$

Under the constraints :

- $\left(\beta\theta + (1-\theta)\left(1-p(x)\right)\right)\Psi^s(u_a) + \left((1-\beta)\theta + (1-\theta)\left(p(x)\right)\right)\Psi^s(u_n) = U$ (14 b)

- $\left(\sigma\omega + (1-\omega)\left(1-p(x)\right)\right)v_a + \left((1-\sigma)\omega + (1-\omega)\left(p(x)\right)\right)v_n + \left(\beta\theta + (1-\theta)\left(1-p(x)\right)\right)u_a +$

$$\left((1-\beta)\theta + (1-\theta)\left(p(x)\right)\right)u_n + (1-\omega)\sigma\Theta_V^s + \beta\theta\Theta_P^s + x + [(1-\omega)\Theta_V^s + (1-\theta)\Theta_P^s]p(x) = u + v \quad (14 \text{ c})$$

for $s = \{NR, NLI, SL, SLC\}$.

(14 c) builds from the decomposition of $\Psi^s(x) + \phi^s(x)$ . This means that the resources used for potential repairs are equal to the present, available resources. We use the Khun-Tucker method to solve this system. Then, for a given $x$ and differentiating the program to $v_a$ and $v_n$, and $u_a$, $u_n$, it appears that $\phi^s(v_a) = \phi^s(v_n)$ (by eliminating multiplicators) for $v_a = v_n = \mu$ and $u_a = u_n = h$, this condition is sufficient and necessary to satistfy (b) and (c). Replacing theses values in the program by $\mu$ and $h$, this one becomes:

$$\underset{x,,,\mu,h}{max}[\phi^s(\mu)] \quad (15 \text{ a'})$$

Under the constraints:

$$\Psi^s(h) = U \quad (15 \text{ b'})$$

$$\mu + h + (1-\omega)\sigma\Theta_V^s + \beta\theta\Theta_P^s + x + [(1-\omega)\Theta_V^s + (1-\theta)\Theta_P^s]p(x) = u + v \quad (15 \text{ c'})$$

By (b'), $\overline{h}$ is determined and it is substituted in (c'). Then:

(16) $\mu = u + v - \overline{h} - \left((1-\omega)\sigma\Theta_V^s + \beta\theta\Theta_P^s\right) - \left(x + [(1-\omega)\Theta_V^s + (1-\theta)\Theta_P^s]p(x)\right)$

As the components of $u + v - \overline{h} - \left((1-\omega)\sigma\Theta_V^s + \beta\theta\Theta_P^s\right)$ are given, the program amounts at minimizing $\left(x + [(1-\omega)\Theta_V^s + (1-\theta)\Theta_P^s]p(x)\right)$. This one is contingent to the liability regime that the regulator has enforced. Indeed, in the bracket expression, we cannot sum indistinctly $\Theta_V^s$ et $\Theta_P^s$. Indeed, this would involve repairing twice as one can verify it by developing the expression : $[(1-\omega)\Theta_V^s + (1-\theta)\Theta_P^s] = \Theta_V^s + \Theta_P^s - \omega\Theta_V^s - \theta\Theta_P^s$.

We will now study this point by examining the optimality conditions of the different liability regimes.

### 3.3 The injurer's irresponsibility

In this scenario, polluters are not liable for their damage. Therefore, the Society as a whole supports the burden of repairs. This irresponsibility, however, does not prevent the regulator to determine the socially first best level of care. In this aim, we define the different steps corresponding to the government's calculation. Hence, the injurers' cost function becomes:

(17) $\Psi^{NLI}(x) = u - x$

The latter are not induced to invest in prevention because, obviously, they are free from any liability in case of an accident and the care level will be null. For the victims, before the accident, the costs of a potential accident depend on their beliefs about it scale:

(18) $\phi^{NLI}(x) = v - (1-\omega)\sigma\Theta_V^s - (1-\omega)\Theta_V^s p(x)$

Hence, the aggregated social utility function is:

(19) $EWS^{NLI} = \Psi^{NLI}(x) + \phi^{NLI}(x) = u + v - x - (1-\omega)\sigma\Theta_V^{NLI} - (1-\omega)\Theta_V^{NLI}p(x)$

And, the socially first-best level of care $x^{NLI} > 0$ is this value for which $\frac{\partial EWS^{NLI}}{\partial x} = 0$

(20)

$$p'(x^{NLI}) = -\frac{1}{(1-\omega)\Theta_V^{NLI}} = -\frac{1}{(1-\omega)\left(\varepsilon\eta\overline{d} + (1-\varepsilon)\eta l + (1-\eta)\overline{l}\right)}$$

Then, the government will have setting up specific regulatory instruments if he wants that $x^{NLI}$ be reached. Note that if the victim feels no preference for ambiguity ($\eta = 0, \omega = 0$), then there are the traditional results obtained under the assumption of neutrality to risk:

(21)

$$p'(x^{NLI})_{|\eta=0,\omega=0} = -\frac{1}{\bar{l}}$$

Where $\bar{l}$ is the expected level of damage.

### 3.4 Strict liability with capped repairs

We present this scheme for heuristic reasons because, here, both victims and polluters bear the burden of reparations. This type of liability governs the nuclear sector and the maritime transportation of oil. Facing the damage hugeness related to the industry activity (risk of radioactivity extended to people and the environment for nuclear or oil spills for the oil sector), the States, on the basis of international conventions, have established liability rules intended to encourage private investment. Strict liability blames the operator of a hazardous activity without need of demonstrating the existence of misconduct, but this liability is limited by a ceiling of the repair. Hence, concerning injurers, the damage scale is limited till a given ceiling $c$ (cap) $c < l$ less than the maximum level of damage. The difference $l - c$ is charged to victims. Hence, their respective payoff functions write as:

#### *a. Injurers*

Limiting the amount of the damage affects the victim's Choquet integral:

(22) $\Theta_P^{SLC} = \alpha\gamma\bar{d} + (1-\alpha)\gamma c + (1-\gamma)\bar{l}$ where $c > \bar{l}$

Then, the program becomes :

(23) $\Psi^{SLC}(x) = u - x - (1-\beta)\theta\Theta_P^{SLC} - (1-\theta)\Theta_P^{SLC}p(x)$ or still :

$\underset{x\geq 0}{Max}\{u - x - (1-\beta)\theta\Theta_P^{SLC} - (1-\theta)\Theta_P^{SLC}p(x)\}$

#### *b. Victims*

Compared to the previous situation where the polluters were liability free, the repairs borne by victims decrease :

(24) $\Theta_V^{SLC} = (1-\varepsilon)\eta(l-c)$

And, after having introduced this expression in the CEU :

(25) $\phi^{SLC}(x) = v - (1-\omega)\sigma\Theta_V^{SLC} - (1-\omega)\Theta_V^{SLC}p(x)$

#### *c. The Regulator*

As previously, by summing (23) and (25), we get the social utility function:

(26) $EWS(x) = \Psi^{SLC}(x) + \phi^{SLC}(x) = u + v - x - (1-\beta)\theta\Theta_P^{SLC} - \omega\sigma\Theta_V^{SLC} - p(x)\Big((1-\theta)\Theta_P^{SLC} + (1-\omega)\Theta_V^{SLC}\Big)$

According to this scenario, the regulator requires a level of protection equal to $x^{*SLC}$, where $x^{*SLC}$ is this value for which $EWS'(x^{*SLC}) = 0$, and,

(27) $p'(x^{*SLC}) = -\frac{1}{(1-\theta)\Theta_P^{SLC}+(1-\omega)\Theta_V^{SLC}}$

The injurer maximizes his payoff for, $x^{SLC}$ where $x^{SLC} < x^{*SLC}$ (This point is proved in appendix 2), hence, his program is:

(28) $\underset{x\geq 0}{Max}\{u - x - (1-\beta)\theta\Theta_P^{SLC} - (1-\theta)\Theta_P^{SLC}p(x)\ \}$

We cans see that this program amounts to minimizing $\{x + (1-\theta)\Theta_P^{SLC}p(x)\}$.

It follows that the regulator has to find the economic tools that induce the tortfeasor to achieve the prevention level $x^{*SLC}$ and no $x^{SLC}$. Here, enforcing negligence is not possible since ceiling the repairs is intended to encourage the producers to invest when the expected level of repairs is too high and may deter the investment in risky activities.

### 3.5 Strict liability vs negligence rule

Here we compare the objective and subjective responsibility regimes. This section shows that uncertainty is not a sufficient condition for proving the absence of Pareto-optimality.

#### *1) Strict liability*

In a strict liability regime, the party who causes the damage is held responsible even without proof of misconduct. The existence of damage and the proximity of the activity or still the activity itself are sufficient to deduce the tortfeasor's liability. To investigate this case, as before, we introduce the polluter's payoff function, the victims' one and we deduce the social utility function..

##### a. *The injurer*

Unlike previous cases, polluters are totally responsible when an accident associated with their activity occurs. Thus, the Choquet integral of the damage is written as:

(29) $\Theta_P^{SL} = \alpha\gamma\overline{d} + (1-\alpha)\gamma l + (1-\gamma)\overline{l}$

And the payoff function is the following Choquet integral of neo-additive capacity:

(30) $\Psi^{SL}(x) = u - x - (1-\beta)\theta\Theta_P^{SL} - (1-\theta)\Theta_P^{SL}p(x)$

*b. The Victims*

Under strict liability, the victims are fully compensated for their losses due to the polluters. Hence, the victims' damage function will be: $\Theta_V^{SL} = 0$ and their payoff corresponds to their initial wealth:

(31) $\phi^{SL}(x) = v$

The victim's payoff function is not affected by the consequences of an accident, when the injurer's wealth is large enough: $u > l$.

*c. The regulator*

As previously, the social utility function is deduced from the aggregation of the agents' utility. Then, the regulator maximizes the following function:

(32) $EWS(x) = \Psi^{SL}(x) + \phi^{SL}(x) = u + v - x - (1-\beta)\theta\Theta_P^{SL} - p(x)\Big((1-\theta)\Theta_P^{SL}\Big)$

We can see that this comes to minimize:

(32') $x + p(x)\Big((1-\theta)\Theta_P^{SL}\Big)$

Here, the regulator requires a level of care equivalent to $x^{*SL}$, where $x^{*SL}$ is this value for which $EWS'(x^{*SL}) = 0$, or still:

(33) $p'(x^{*SL}) = -\frac{1}{(1-\theta)\Theta_P^{SL}}$

$x^{*SL}$ is the care effort which is socially optimal and desired by the regulator. For its part, the injurer maximizes his payoff for, $x^{SL}$ where $x^{SL} = x^{*SL}$. This result deduces from his program $\underset{x\geq 0}{\text{Max}}\{u - x - \beta\theta\Theta_P^{SL} - (1-\theta)\Theta_P^{SL}p(x)\}$ in which, by the same argument than above, it appears that the injurer must minimize the following accident costs:

$$x + p(x)\Big((1-\theta)\Theta_P^{SL}\Big).$$

Hence, the regulator and the injurer minimize the same function. We deduce the following proposition:

**Proposition 1 :** *Under Knightian uncertainty and under strict liability, then a social optimum will be achieved with injurers wealth higher than damage* ($u > l$)*: the victims will be fully reimbursed for their losses and the level of prevention effort will be a first best level.*

**Proof:** The proof follows from the above argument.

**Remark 1**: This result is similar to the one reached with the basic unilateral accident model when, both, regulator and injurer are risk neutral (see Shavell 1987 (b)). Here, the agents' ambiguity aversion (polluters and victims) is not a hurdle to the implementation of the first-best. This is because the regulator is not dictatorial considering the agents' preferences that he aggregates

### *2) The negligence rule*

Under negligence, the formal results are not different from the ones got under strict liability. The payoff functions of both categories of agent (injurers and victims) are similar to the case of strict liability as shows it the following proposition:

**Proposition 2:** *Under Knightian uncertainty and negligence rule, then the standard of due care equals the first best-level.*

***Proof :*** The proof is similar to Shavell (1984). Hence, to avoid the loss of a part of his wealth in case of an accident, the injurer supplies a care level higher or equal to the first-best:

(34) $\Psi^{NR}(x) = u - x - (1-\beta)\theta\Theta_P^{NR} - (1-\theta)\Theta_P^{NR}p(x)$ if $x^{NR} < x^{*NR}$

And,

(34') $\Psi^{NR}(x) = u - x,$ if $x^{NR} \geq x^{*NR}$.

The tortfeasor cannot invest less than $x^{*NR}$, but, also, he cannot supply a higher effort. Consequently, the socially first best level is $x^{*NR}$. For determining $x^{*NR}$, it is sufficient to verify that :

$$\Theta_P^{NR} = \Theta_P^{SL} = \alpha\gamma\overline{d} + (1-\alpha)\gamma l + (1-\gamma)\overline{l}$$

Consequently:

(35) $p'(x^{*NR}) = p'(x^{*SL}) = -\frac{1}{(1-\theta)\Theta_P^{NR}}$. And, naturally : $x^{*SL} = x^{*NR}$, because, here (11) (d).

From this result, and from proposition 1 and 2, the following proposition 3 deduces:

**Proposition 3:** *Under Knightian uncertainty, whatever the current liability regime, then the first-best effort will be the same:* $x^{*SL} = x^{*NR}$.

**Remark 2:** Propositions 1 to 3 contradict the recent literature for which radical uncertainty causes inefficiency. When the agents' wealth covers the damage costs, whatever the liability regime (strict liability or negligence), the required socially first-best care is identical. As the polluter implements the optimal care effort, then the regulator will not have to define incentives to reach this goal. Inefficiency means the difference between the first rank security level determined by the regulator and the injurer's optimum level of safety. Thus, in this context, inefficiency does not come from uncertainty, but from the tortfeasor's inability of fulfilling his repair's commitment. Several factors explain this situation. The first one is the definition of legal ceiling for repairs. Another possibility comes from the situation where the injurer becomes "judgment-proof" as described by Shavell (1986), Summers (1983). This state can be deliberately organized by the polluter (Van't Veld and ali ((1997), van 2006). In all cases, the ineffectiveness of the equilibrium solution is verified. In our representation this situation is illustrated in Section 3.4 when repairs are bounded by an institutional ceiling. Shavell (1982) shows that when the polluter feels risk aversion, then, faced with a neutral regulator vis-à-vis risk, the solution is not effective. He highlights, that the necessary prevention level will be more important (Shavell (1982)) that involves more wealth dedicated to repairs.

**Remark 3:** In the standard unilateral accident model, the court is poorly defined and remains in the background. In fact, with a dictatorial, neutral to risk regulator, the court (assuming it does not make mistakes) has to follow the regulator's position. Otherwise, this involves that the judge owns a specific utility function (e.g. showing a risk aversion) that differs from the regulator one. Under negligence, there would be a potential divergence level between the court and the regulator concerning the assessment of the socially optimal care level. Hence, the polluter could legitimately feel perplexity wondering about the effective level of care to implement. This would lead to indeterminacy for the regimes based on negligence: should he follow the regulator or should he forecast the judge's one? To ask the question is to answer it: under dictatorial governance, the court cannot diverge from the regulator choice. Then, court is reduced to a mere verification body. Under radical uncertainty and a no-dictatorial regulator, the court may have an independent existence despite the similarities with the standard case. Indeed, the court may accept not having a dissent concerning the first-best care level: it legitimately may accept the regulator's predominance since the social utility function reflects the agents' preferences. If by chance the court had a different opinion, this would mean that it substitutes its own logic to the regulator's one and its decisions can be considered as arbitrary. However, the court could dispose of new information that has not been previously transmitted by one of the parties to the regulator. This last possibility deserves discussion, but that will take us too far.

## 3. Conclusion

Simple in its basic formulation, the unilateral accident model is far from having revealed all its potential. The question of the regulator’s utility function is central. When the regulator is dictatorial, reaching the first-best level of care involves that his preferences are consistent with the injurer ones (both neutral to risk). With agents that feel aversion to risk or/and ambiguity the solutions are not optimal. This is the current result on which issued the literature of radical uncertainty since Teitelbaum (2007). In our opinion, this result comes from the specific assumption that the contemporaneous literature made on the regulator.

Indeed, things become different when the regulator aggregates the agents’ utility functions and this is the point the model focused on. It extends uncertainty to victims and spreads on two structural levels: the distribution of accident probabilities on the one hand and the scale of major damage on the other one. With a non-dictatorial benevolent regulator, uncertainty does not induce the inefficacy of the equilibrium solution. Furthermore, the liability regimes, strict liability and negligence, can be considered as equivalent as in the standard unilateral accident model without ambiguity. In fact, inefficiency of the social care level comes from the level of the polluters’ wealth. Indeed, when this wealth is insufficient to cover the damage costs, it causes a difference between the social first-best of care and the private one. This discrepancy between wealth and damage (damage costs higher than injurer’s wealth) is the main cause of the social inadequacy of the prevention level rather than radical uncertainty. This is what literature called as the "judgment-proof" question.  May one direct future research will be to assess which liability regime fits better under the injurer’s judgment-proof situation.

Starting from this basis, many questions remain to examine as, for instance, the errors made by the Courts, or more precisely the differences between the regulator’s assessment and the Courts’ estimations. This issue is particularly sensitive for the comparison between strict liability and negligence. Another less legalist question deals with issues relating to the risk assessment the government over the potential injurers’ estimates.

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## Appendix 1

## The concept of neo-additive capacity.

More generally, the criticism of the expected utility foundations began in the midst of last century when, first, Allais (1953) criticized the Savage's independence axiom. Furthermore, Ellsberg in 1961 showed that the Savage's preference preorder leads to the paradoxical situation in which the sum of probability on uncertain events differs from one[4]. To explain this, let us see a schematic example. Hence, consider a player and two urns that contain, each, blue and red balls. The player wins x dollars when he draws a blue ball from one urn. In one of these, he knows the probability of drawing a red or a blue ball, while in the other urn this proportion is unspecified. Before the ball draw, the player must choose the urn in which he will draw the ball. Consequently, if he chooses the urn in which the proportion of blue and red balls is known, he implicitly considers that, with this urn, the odds of winning are greater than with the other one. Thus, he marks an aversion to ambiguity. Hence, if the odds of winning are 45% with the urn in which probabilities are given, he considers that the probability of winning is less than 45% if choosing the other one. By doing so, it appears that the sum of the probabilities for a given event is greater than 1.

Schmeidler (1989) systematizes Ellsberg's approach by using Choquet's integrals as substitute to the Savage Expected Utility theory (SEU). For modern ambiguity theory, a non-additive probability or "capacity" represents the agents' beliefs about the likelihood of events. The agents maximize a utility function based, not on the sum of weighted utility indices, with weights that sum to 1 as in the theory of Savage, but for a sum greater than 1 which represents a Choquet integral. It is admitted that according the integral shape (concave or convex) the agent expresses optimism (concavity due to super-addivitivity) or pessimism (sub-additivity)[5]. Schmeidler's approach hardly lends itself to manageable extensions. However, Chateauneuf, Eichenberger and Grant (2007) (CEG) performed this task by developing the concept of neo-additive capacity. Due to its characteristics, this concept allows integrating the

[4] See Teitelbaum (2007) for a complete review.
[5] Voir Teitelbaum(2007).

contributions of experimental economics in the decision field[6]. Indeed, this capacity is additive on non-extreme values but non-additive on maximum and minimum values. This means that, for example, in bets situations, the "real" persons do not behave as predicted by the expected utility theory. Indeed, they tend to overestimate the probability of higher earnings while generally these one are close to 0 (the case of national lotteries) and tend to underestimate the probabilities of losses for low earnings (see Camerer and Weber (1992) Gonzales and Wu (1999) or Abdellaoui (2000)).These results are illustrated by the well-known inverted S-shaped curve. Appendix 1 of this article briefly presents the mathematical foundations of this approach.

We do not propose here a full formal mathematical presentation. The interested lector may refer to the clear exposition of Chateauneuf, Eichberger and Grant (2007) (CEG(2007) in the following).

A capacity is an extension of a probability. It is a function $\tau(p)$ that assigns real numbers to events $\mathcal{E}$, where $\mathcal{E}$ is the set built from the set $\mathbb{S}$ of the states of nature. A capacity fulfills two conditions. First, for all $E, F \in \mathcal{E}$, and $E \subseteq F$, then $\tau(E) \subseteq \tau(F)$ as monotonicity condition and, second, as normalization conditions, $\tau(\emptyset) = 0$ and $\tau(\mathbb{S}) = 1$.

The best way to integrate capacities is the Choquet integral. To do that, it is assumed that exists a simple function of finite range $f$ that takes values $\mu_1 \geq \mu_2 \dots \geq \mu_n$. A Choquet integral of a simple function $f$ with respect to a capacity щ$(.)$ is defined as:

$$V(f/\tau) = \sum_{\mu \in f(\mathbb{S})} \mu[\tau(\{s/f \geq \mu\}) - \tau(\{s/f > \mu\})] \quad (1A)$$

Through the concept of neo-additive capacity the Choquet integral overweight high outcomes if the capacity is concave or overweigh low income if the capacity is convex. Convexity of a capacity is verified by the following relationships:

$\tau(E \cup F) \geq \tau(E) + \tau(F) - \tau(E \cap F)$ (and concave in the opposite situation).

Applying this to our model, we consider that the polluter and the society cannot assess with certainty the exact value of a maximum damage. Let be $\mathcal{E}$ the finite set of states to which correspond the catastrophic events $\mathcal{A}$ ($\sigma$-algebra of $\mathcal{E}$). We consider a finite set of outcomes ( $A \subset \mathbb{R}$) and let $\Phi = \{f: \mathcal{E} \to A\}$ be a set of simple functions from states to outcomes which correspond to simple acts and takes on values $a_1 \geq a_2 \dots \geq a_n$.
The polluter is gifted with a Choquet objective function which corresponds here to an expected cost function. His beliefs on the level of damage correspond to a neo-additive capacity ($\mu$) based on ($p$). Hence, the operator will form beliefs about the level of the damage. This is a supplementary uncertainty. We can define now the neo-additive capacity. To do that let us consider that the $\sigma$-algebra $\mathcal{A}$ is partitioned in three subsets that we present and characterize (for a more complete information see CFG (2002, 3).

- The set of null events $\mathcal{N}$, where $\emptyset \in \mathcal{N}$ and for $G \subset H$, and $G \in \mathcal{N}$ if $H \in \mathcal{N}$.
- The set of "universal events" $\mathcal{W}$, in which an event is certain to occur, (complement of each member of the set $\mathcal{N}$).
- The set of essential events, $\mathcal{A}^*$, in which events are neither impossible nor certain. This set is composed of the following:

$$\mathcal{A}^* = \mathcal{A} - (\mathcal{N} \cup \mathcal{W})$$

Before going further, we define the following capacities $\nu$ (see appendix):

$\nu_0(A) = 1$ if $A \in \mathcal{W}$ and 0 otherwise and $\nu_1(A) = 0$ for $A \in \mathcal{N}$ and $\nu_1(A) = 1$ otherwise.

Furthermore, we define a finite additive probability $p(.)$ such that $p(A) = 0$, if $A \in \mathcal{N}$ and 1 otherwise.

**Definition 1:** Let $\lambda$, $\gamma$ that belong to a simplex $\Delta$ in $\mathbb{R}^2$, ($\Delta := \{(\alpha, \beta) / \ \alpha \geq 0, \beta \geq 0, \alpha + \beta \leq 1\}$), a neo-additive capacity $\mu$ based on the distribution of probability $p(.)$ is defined as:

$$\mu(A / p, \lambda, \gamma) = \begin{cases} 0 \ for \ A = \emptyset \\ \lambda \nu_0(A) + \gamma \nu_1(A) + (1 - \gamma - \lambda) p(A) \ for \ \emptyset \subsetneq A \subsetneq \mathcal{E} \\ 1 \ for \ A = \mathcal{E} \end{cases} \quad \text{(2A)}$$

A neo-additive capacity is additive on non-extreme outcomes. Here $p$ corresponds to the probability of a major accident of a given scale. This is a common belief and $(1 - \gamma - \lambda)$ represents the degree of confidence of the agent in this belief. We will give below, after the presentation of the Choquet integral of the neo-additive capacity, more complete explanation on the concept of optimism.

Then, we can define the Choquet integral which is a weighted sum of the minimum, the maximum and the expectation of a simple function $f: \mathcal{E} \to \mathbb{R}$ as it is expressed in the following relationship:

$$V(f / p, \lambda, \gamma) = \lambda . \inf(f) + \gamma . \sup(f) + (1 - \gamma - \lambda) E_p(f) \quad \text{(3A)}$$

Where $E_p(f)$ is the expected value of the expected costs of a major accident, and from the linearity of the Choquet integral with respect to the capacity, we define $V\big(f / \nu_0(.)\big) = \inf(f)$ and $V\big(f / \nu_1(.)\big) = \sup(f)$, (proof see CFG(2002, 3) and CFG(2006, 3).

Then for $e \in \mathcal{E}, f(e) = a$, we put, $f(e_1) = \sup(f) = a_1 = l$ and $f(e_n) = \inf(f) = a_n = \overline{d}$. As, $p(.)$ is a finitely additive probability distribution on $\mathcal{A}$, we define $E_p(f)$ as:

$$E_p(f) = E_p(a) = \int_{\overline{d}}^{l} a \, p(a) da \quad \text{(4A)}$$

Taking into account these factors, the Choquet integral writes now:

$$V_p = \lambda . \overline{d} + \gamma l + (1 - \gamma - \lambda) E_p(a) \quad \text{(5A)}$$

Hence, if $\gamma = \lambda = 0$, we find the usual expected utility. With $1 \geq \gamma > 0, \lambda = 0$, the subject is waiving between the lowest value and the expected value of the function. That corresponds to pessimism because the operator cannot consider that $l$ occurs with sufficiently high probability. Then, optimism is induced by $\gamma = 0$, $1 \geq \lambda > 0$.

In order keeping a correspondence with the Teitelbaum (2007)'s analysis, we make the following change of variable that corresponds to the treatment of CEG (2007) in their paper :

$\lambda = \delta\alpha, \gamma = \delta(1-\alpha)$, then we can check that $1-\gamma-\lambda = 1-\delta$ with $\delta, \alpha \in (0,1)$
The neo-additive capacity is then:

$$\mu(\,A\,/\,\,p, \delta, \alpha) = \begin{cases} 0\, for\; A = \emptyset \\ \delta\alpha v_0(A) + \delta(1-\alpha)v_1(A) + (1-\delta)p(A)\, for\; \emptyset \subsetneq A \subsetneq \mathcal{E} \\ 1\, for\; A = \mathcal{E} \end{cases} \quad (6A)$$

Or, still, for $\emptyset \subsetneq A \subsetneq \mathcal{E}$

$$\mu(\,.\,) = \delta\alpha\overline{d} + \delta(1-\alpha)l + (1-\delta)p(A) \quad (7A)$$

we get then the neo-capacity's Choquet Integral:

$$V_p = \delta\alpha\overline{d} + \delta(1-\alpha)l + (1-\delta)E_p(a) \quad (8A)$$

The precise meaning of the weight $\delta$ (aversion for ambiguity) and $\alpha$ (degree of optimism) is made in the paper.

## Appendix 2

Proof that : $x^{SLC} < x^{*SLC}$.

This proof is classical. Let us check that $x^{SLC}$ verifies $\Psi'^{SLC}(x^{SLC}) = 0$, and $p'(x^{SLC}) = -\frac{1}{(1-\theta)\Theta_P^{SLC}}$ .

For $(1-\omega)\Theta_V^{SLC} \geq 0$, $(1-\theta)\Theta_P^{SLC} + (1-\omega)\Theta_V^{SLC} \geq (1-\theta)\Theta_P^{SLC}$, and,

$\frac{1}{(1-\theta)\Theta_P^{SLC}+(1-\omega)\Theta_V^{SLC}} < \frac{1}{(1-\theta)\Theta_P^{SLC}}$ , for $\theta \neq 0$

Consequently :

$p'(x^{*SLC}) = -\frac{1}{(1-\theta)\Theta_P^{SLC}+(1-\omega)\Theta_V^{SLC}} > \; p'(x^{SLC}) = -\frac{1}{(1-\theta)\Theta_P^{SLC}}$ , $p'(x) < 0$

However, as $p''(x) > 0$, , $p'(x)$ is an increasing function, $p'(x^{*SLC}) > \; p'(x^{SLC})$ involves that $x^{*SLC} > x^{SLC}$.

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