Real Firms, Transaction Costs and Firm Development: A Suggested Formalisation

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Abstract

The motivation of this discussion is threefold: to integrate transaction costs (TCs) into a standard model of the firm; to examine the interaction between organisational factors (i.e. TCs) and standard demand-cost factors; and to analyse key propositions of transaction cost economics with the general model. Two sets of results are derived. First, when analysis is based on significant interaction between organisational effort and production costs two possible organisational solutions can exist. First we have a “normal” relationship that the existence of small firms is subject to a threshold effect for transaction complexity. Secondly large firms can develop because of interactions between organisation effort and marketing and production costs. A second key result concerns strategies to shift from small to large solutions that can be based on either “small steps” or “developmental leap”. The viability of these alternatives is shown to depend on transaction complexity that affects the transition costs involved. In short these findings collectively indicate that analysis of the interaction between organisational and technical aspects of the firm using a formal method does indeed add value in terms of our understanding.

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JEL codes: D23, D47, L22
Introduction
In his Nobel Prize acceptance speech Oliver Williamson (2009) outlines the nature of his transaction cost (TC) project and suggests areas in need of progression. One such area involves formalisation of theory. He claims (p471)

Transaction cost economics is sometimes criticized because it has not been fully formalized, to which I have three responses: transaction costs economics, like other theories, has undergone a natural progression; full formalization is a work-in-progress; and premature formalization runs the risk of a disconnection with the phenomena.

The current paper can be understood as part of this work-in-progress. The key objectives are threefold:
1. Integrate TCs into a standard model of the firm.
2. Examine the interaction between organisational factors (TCs) and standard demand-cost factors.
3. Analyse key propositions of transaction cost economics (TCE) with the general model.

This paper is, of course, not the only attempt at formalising TCE and organisational aspects of the firm: Grossman and Hart (1986), Hart and Moore (1990), Tadelis (2002), Bajari and Tadelis (2001) are examples. But a specific characteristic of the current discussion is an emphasis on “real” firms, as highlighted in the title. This builds on the TC tradition of avoiding “disconnection with the phenomena”, to use Williamson’s formulation in the above quotation. This emphasis on real firms builds on the formulation suggested by Coase (1937, 1993) that we need a “realistic” theory of the firm rather than the “blackboard economics” (Coase 1991) that characterises much writing. For current purposes we view real firms as being both institutional and technical. The institutional firm characteristically concerns itself with issues such as internal structure, organisation and boundaries. The technical firm analyses behaviour in particular market contexts in terms of demand-cost interaction. Both perspectives on the firm are relevant for analysis because the market context and demand-cost interaction can interact with institutional factors such as organisation and boundaries.

In abstract terms the discussion here can be viewed as an extension of a standard transaction cost perspective that views firms as bundles of transactions. The resulting
emphasis on exchange with this perspective tends to downplay production and other technical matters and their interaction with contracting-exchange. From a standard transaction cost perspective the characteristics of the real goods or services being produced are independent of the institutional analysis. In Williamson’s (1985, p 22) words: “Holding the nature of the good or service to be delivered constant, economizing takes place...”. It follows that an important driver of the current discussion is that interconnections between institutional and technical factors are (potentially) important, hence objective two of the three objectives for this paper highlighted above. It will be shown below that modelling interconnections between institutional and technical aspects of the firm qualifies some accepted conclusions. But because of the potential complexity of the interactions a formal method is useful a claim that echoes Garrouste and Saussier (2005) and Masten and Saussier (2002).

The rest of the discussion is organised as follows. In the next section a number of general issues involved with modelling real firms are set out and in particular the formal analysis of transaction costs. Following this the analytical framework is developed involving a single firm analysis in which profit maximising output and organisation activity are chosen. Following derivation of key results, and to facilitate further analysis, particular parameterisations are used based on a constant elasticity unit production cost function. This formulation allows particular conclusions to be drawn that can both reproduce aspects of standard TC logic but also deviate from this logic depending on parameter values. In the penultimate section the solutions predicted by the analysis are further examined. Firm viability issues are considered in terms of a non-negative profitability constraint. This qualifies but does not fundamentally change earlier analysis. In addition the framework is shown to predict two firm types: the existence of small firms and the development of large firms. The first type appears consistent with a Williamson TC logic. The second type is closer to a Chandler-type logic. The possible shift from small to large firm solutions can involve “small steps” or “developmental leap” strategies with the relevance of either depending on adaptation costs that in turn depend on transaction complexity. Finally conclusions are drawn.
Modelling real firms: general issues

The technical analysis presented here will be based on a single firm model apart from more general discussion and concluding comments. All relationships are assumed continuous and (apart from demand functions and organisational costs) twice differentiable. We assume linear demand; there is no real loss of generality here. TCs are viewed in the standard way as the costs of search, negotiation and policing that accompany any transaction i.e. exchange of a good or service. This exchange need not be market based but can also be exchange within a firm. To render the analysis tractable we build on the early TC tradition (Coase 1937, Williamson 1975) that views the governance choice as between markets and intra-firm hierarchies rather than the complexity of institutional forms involving “intermediate” governance structures analysed by more recent work (for example Williamson, 1985; Künneke, Groenewegen, and Ménard, 2011; Ménard, 2004).

The extent of TCs is modelled in terms of organisational effort. To gain more information, and use this information during negotiation or policing, any firm requires additional organisational factor inputs. One aspect of this framework involves deriving profit maximising equilibrium effort given the fundamentals of the model. This level of effort can then be used to define equilibrium transaction costs. We therefore ignore dynamic and developmental issues and instead rely on a comparative static framework. What is not considered are possible dynamic effects in which the extent of transaction costs are allowed to have a feedback impact on organisational effort. For similar reasons we also ignore issues, such as those suggested by Penrose (1959), in which firm growth and evolution can be based on managerial excess capacity. In this dynamic context acquisition and use of information need not be based on additional organisational factor inputs because of managerial excess capacity.

The “organisational technology” defines the relationship between effort and TCs. This technology is summarised in terms of the first derivative of the relationship (see below). Using standard TC theory the link between effort and TCs will depend on the fundamental characteristics of any transaction, perhaps most notably (using Williamson, 1985) uncertainty and asset specificity. With high levels of the latter characteristics the first derivative of the TC relationship will be large. For example, an increase in organisational inputs used for search activity will involve a greater
increase in TCs (with constant organisation input prices per unit) if the search activity is complex because of uncertainty and/or asset specificity factors.

The linkages between organisational factors and standard demand-cost factors are modelled here in terms of firm revenues and average production costs being endogenous to organisational effort. For example, greater search or negotiation can result in a more effective pricing policy i.e. we can expect a non-negative relationship between output price and effort (with the assumed single product firm). For average production costs we assume a non-positive relationship with effort. The reasoning here is that, for example, extra effort might result in more effective management of production, and hence greater technical efficiency and lower average production costs. Alternatively extra effort might control post-contractual opportunism more effectively in input markets. In abstract terms we can think of a notional production cost frontier that is asymptotically approached with extra effort. From a transaction cost perspectives (Williamson, 1985) the difference between actual and potential production costs are viewed as misalignment costs because of incomplete contracts. The profit maximising level of effort analysed here can therefore be viewed as the analogue of an efficient level of transaction costs used to control misalignment costs in an optimal manner.

Allowing for both production activity and organisational factors implies that we have two measures of firm size: physical activity (measured for example as real output) and transaction costs, with the latter assumed to have a monotonically increasing relationship with its organisational “input” i.e. effort. This implies that the standard institutional analysis of the “existence of firms” can be recast here as a requirement for positive organisational effort. Hence the existence of real firms (positive effort and real output), in the modelling presented below, is both an organisational and technical feature.

Obviously any real firm undertakes many organisational tasks that cover the management of output markets, intra-firm activity and input markets. In addition there will be characteristic substitutability and complementarity between these tasks. To render the modelling tractable we assume that these various managerial tasks are undertaken in fixed proportions with, in addition, no substitution between
organisational human and non-human inputs being possible. This fixed proportions organisation technology assumption allows us to analyse a “basket” of organisational tasks in which the internal characteristics of the basket are fixed although the whole basket may change size in a continuous manner. This simplification allows us to create an aggregate measure of transaction costs that is simply the sum of the various specific organisational costs (that are not modelled). We therefore have a single measure of organisational activity that can be applied in different contexts. In addition the size of the organisation “basket” can be viewed as the organisation capacity if the firm.

To model organisational activity we view our aggregate measure of transaction costs ($C_T$) as a function of organisation effort ($e$), an index of organisation input prices ($p_O$) and an index of transaction specific factors ($t$). In general terms:

$$C_T = f(e, p_O, t)$$  \hspace{1cm} (1)

Effort can be measured as organisational input per unit time. In principle this is unbounded from above. In addition both $C_T$ (the organisational “output”) and $e$ (the “input”) provide measures of organisation size or capacity. Throughout the discussion we assume $p_O$ constant. The index $t$ can be conceptualised as a ranking of possible exogenous features that determine transaction complexity. Greater complexity implies, ceteris paribus, greater transaction costs with constant $e$ and $p_O$ i.e. there is a direct effect of $t$ on $C_T$. In addition, increasing complexity will increase the costs associated with search, negotiation and policing activity for any given change in effort i.e. there is also an indirect effect of $t$ on $C_T$. Using standard TC reasoning this complexity will depend on (most notably) asset specificity and transaction uncertainty.

A corollary of the fixed proportions “organisation basket” assumption made above is that organisational technology can be viewed as linear along the lines suggested in Figure 1. With constant $p_O$, transaction costs are depicted for two levels of transaction complexity $t_2 > t_1$. Given this conceptualisation, it follows that increasing transaction complexity will increase $f'(e)$. This reasoning is used below where $f'(e)$ is used as an exogenous determinant of equilibrium effort. In addition increasing $f'(e)$ is viewed as reflecting greater transaction complexity and furthermore can be used an indicator of underlying transaction characteristics.
We can link this general discussion of the modelling presented here to more conventional TC theory. TCE uses a characteristic comparative institutional approach. Within a simple markets-hierarchies framework this can be presented in the following way. For the production of a particular good or service indentify all necessary transactions or organisational tasks. Each transaction can be undertaken either within the firm using hierarchies, for which there is a transaction cost $c_H$, or external to the firm using markets, for which there is a transaction cost $c_M$. For each transaction the difference $c_M - c_H$ can be computed. Using a standard formulation, adopted here, the size of any difference depends on transaction complexity. Starting from the largest difference, an efficient firm will then internalise transactions until $c_M - c_H = 0$. If then a change in contracting conditions takes place, i.e. transaction complexity changes, that affects all transactions equivalently, an efficient firm will change its boundaries, involving more or less market activity, until $c_M - c_H = 0$ is once again attained. If the change in transaction complexity affects the various transactions differently, the ranking of the difference $c_M - c_H$ changes but the end result is the same: an increase or reduction in internalisation. In terms of the formulation used here, a change in transaction complexity affects organisation capacity as measured by effort. We can note that there can be some low level contracting complexity where internal firm organisation is not rational i.e. $c_M - c_H$ is non-positive for all transactions, even if this low level of complexity exists only theoretically rather than in practice.

The framework developed here has the following obvious differences to a comparative institutional logic. We assume a basket of organisational tasks that can vary in size but not internal structure. This facilitates the technical analysis based on profit maximisation with two choice variables: physical output and organisational effort. An implication is that this “organisational basket” approach avoids any requirement for a comparative institutional method. But we are still able to draw the key conclusion that with minimal contracting complexity the existence of firm organisation is not rational. Whereas above some threshold $f'(e)$ a viable organisational solution exists. In addition, and depending on parameter values, because of inter-linkages between organisational and technical aspects of the firm two
effort solutions can be derived. The first defines the existence of small firms the second defines the development of large firms. Both firms types are affected by transaction complexity, i.e. $f'(e)$, as would be expected by TCE. Furthermore, possible strategies that can be used to shift between these firm types are influenced by $f'(e)$. These strategies are summarised as “small steps” or “developmental leap”, with increasing $f'(e)$ eliminating the possibility of the former strategy. Hence this formal analysis of real firms suggests both a partial departure from standard TC theory and a development of the analysis of the firm.

A single firm framework with transaction costs
This section develops a framework in which a firm maximises profit with respect to physical output ($x$) and organisation effort ($e$). We use a linear inverse demand function in which selling price is determined by output and organisational effort:

$$\delta - \beta x + e$$

In (2) $\alpha$, $\beta$ and $\delta$ are positive constants. The logic here is that stated above: greater search, negotiation and/or policing activity can increase selling price without a reduction in demand. Or with a simple reformulation of the function: increasing organisational effort can increase demand without a lower selling price. We can view the effort effect in (2) as marketing effort, in which case the assumed linearity of the relationship is consistent with the Schmalensee (1972) evidence that for television and newspapers there is probably constant advertising costs; although this effect is likely to be industry specific (Sutton, 1991). More generally, the formulation in (2) can be viewed as a first order approximation of an unknown marketing response function.

We define average production costs ($a$) that are endogenous to organisational effort as well as physical output. Total costs ($C$) are the sum of total production and organisation costs:

$$C = xa(x,e) + f(e)$$

(3)

The nature of this cost function is further discussed below. The profit function is now given by:

$$\pi(x,a(x,e),e) = ax - \beta x^2 + \delta e - xa(x,e) - f(e)$$

(4)

The firm has two choice variables: output and effort. The first order conditions for maximum profit are given by:
\[
\frac{\partial \pi}{\partial x} = \alpha - 2\beta x + \delta e - a(x,e) - xa_{x}(x,e) = 0
\]  
(5a)

\[
\frac{\partial \pi}{\partial e} = \delta x - xa_{x}(x,e) - f'(e) = 0
\]  
(5b)

Based on earlier discussion we can offer the following interpretation of the partial derivatives:

\( a_{e}(x,e) \leq 0 \) defines the organisational effort impact on unit production costs.

\( a_{x}(x,e) \leq > 0 \) is an indicator of production scale effects: increasing/constant/decreasing returns to scale imply this derivative is negative/zero/positive.

\( f'(e) \geq 0 \) is an indicator of transaction complexity.

Using (5a) we can define a condition for real output:

\[
x = \frac{\alpha - a(x,e) + \delta e}{2\beta + a_{x}(x,e)}
\]  
(6a)

The formulation in (6a) shows that profit maximising output is a positive function of organisational effort. It is straightforward to show that we might expect the effort-output relationship in (6a) to have a shape defined by \( \partial x / \partial e > 0 \) and \( \partial^2 x / \partial e^2 < 0 \).

Initially assume constant returns to scale in production at all output levels i.e. \( a_{e} = 0 \).

Using (6a):

\[
\frac{\partial x}{\partial e} = \delta \frac{a_{x}}{2\beta} - \frac{a_{e}}{2\beta}
\]

By assumption \( a_{e} \) is non-positive hence \( \partial x / \partial e \) is non-negative for all parameter values. Defining the second derivative:

\[
\frac{\partial^2 x}{\partial e^2} = -\frac{a_{ee}}{2\beta}
\]

Subject to \( a_{ee} \) being positive the second derivative is negative. Discussion presented below uses a particular formulation for average production costs for which the impact of effort is defined by a constant elasticity. This formulation has the characteristic that \( a_{ee} \) is positive.
This general depiction of (6a) is shown in Figure 2. Later discussion links profit maximising effort to transaction complexity. Hence the analytical logic implies that complexity determines organisational effort and in turn effort determines physical output. Although this discussion is based on the case of \( a_s = 0 \) the results are more general. Consider the case of increasing returns to scale in production but the \( a_s < 0 \) being constant. Subject to \( 2\beta > a_s \) this case shifts the curve up in Figure 2. More specifically the condition \( 2\beta > a_s \) is related to the required concavity with respect to \( x \) whereby \( \beta > a_s \), a second order requirement for a profit maximum. For the case of diminishing returns in production the curve in the diagram will shift down. In addition we can point out that any positive level of effort produces positive profit maximising output, subject to the second order requirement of \( \alpha \) greater than average production costs. Finally we can point out with respect to Figure 2 the desirable property that an increase in overall market size (\( \alpha \)) will shift up the curve but not change the slope defined by the derivatives.

Figure 2 here, see end.

This discussion indicates that any solution for the level of physical output requires a solution to organisational characteristics (i.e. effort). Using (5b):

\[
x = \frac{f'(e)}{\delta - a_s(x, e)}
\]  
(6b)

Combining (6a) and (6b) we define organisational effort in terms of the fundamentals of the model:

\[
e = f'(e)\left[\frac{2\beta + a_s(x, e)}{\delta^2 - \delta a_s(x, e)} - \frac{\alpha - a(x, e)}{\delta}\right]
\]  
(7)

The condition defined in (7) shows a relationship between \( f'(e) \), interpreted as contract complexity, and organisational effort, with the nature of the relationship being determined by the fundamentals of the model. The second term on the right hand side of (7) defines a threshold impact of \( f'(e) \) on effort; this follows from a basic firm viability condition. At low \( f'(e) \) no positive organisational activity is correspondingly possible; a feature of (7) that appears consistent with standard TC
theory as discussed above. But the “slope” impact of \( f'(e) \) on effort can be positive or negative.

Further analysis of (7) will restrict itself to the case of constant returns in production at all output levels i.e. \( a_x = 0 \). This allows the discussion to concentrate on the interaction of production and organisational matters rather than production directly. In addition discussion will consider the interaction of two factors: potential market size \((\alpha)\) and the extent to which production costs respond to effort \((a_e)\). If we adopt a market power interpretation of \( \alpha \), the marrying of these two factors (competition and the effect of effort on costs) is logical. Output market dominance can only be maintained if there are defences against new firm entry. New firms will have entry opportunities if production cost advantages can be developed based on organisational activity (here summarised as effort). When average production costs are responsive to effort, new firm entry, and hence competitive markets, are likely. Entry in such conditions will be based on absolute cost advantages from organisational effort. With firm entry our single firm model is more accurately one of monopolistic competition rather than monopoly with entry reducing the size of \( \alpha \). We will see that equilibrium conditions for \( e \) depend on these two factors: \( \alpha \) and \((a_e)\).

As average production costs and its first derivative with respect to effort are themselves functions of effort, we use the assumption of constant returns and re-write (7) in the following form:

\[
\frac{[\delta - a_x(x,e)][\alpha - a(x,e) + \delta e]}{2\beta} = f'(e) \tag{7a}
\]

It is clear from (7a) that the relationship between contract complexity (i.e. \( f'(e) \)) and organisational activity is a non-trivial function of effort. Because of this complexity, further discussion of equilibrium effort will be based on a particular formulation. For all cases a constant elasticity unit production cost function with constant returns in production is used:

\[
a = x^0 e^e \tag{8}
\]

In (8) \( \varepsilon \) is non-positive. Using (8) the formulation in (11a) can be re-written:
\[
\frac{[\delta - \alpha e^{(\varepsilon-1)\varepsilon}][\alpha - e^{\varepsilon} + \delta e]}{2\beta} = f'(\varepsilon)
\]  
(7b).

The constant elasticity formulation in (8), or (7b), suggests that there is no “control loss” as organisation capacity (e) increases i.e. \(\varepsilon\) is unchanged at different levels of effort. This feature is somewhat different from organisational discussion that suggests control loss due to incentive, motivation and similar issues as organisational size increases. As a robustness check on this characteristic of (8) the results reported below were reproduced with a semi trans-log formulation:

\[
a = x^0 e^{(\varepsilon_1 + \varepsilon_2)}
\]  
(8a)

In this latter formulation \(\varepsilon_1\) is non-positive and \(\varepsilon_2\) is strictly positive. Subject to the condition that the overall exponent on effort is negative, the results are structurally the same as those reported for (8) and hence are not reported here. In addition the discussion of firm viability, in the next section, is relevant here. This suggests that at some large level of effort profitability can become negative and hence places a constraint on organisation size. So, while there is no direct control loss in (8) there is an indirect effect that results from the interaction of technical and organisational factors. It is perhaps more significant that a form of control loss can be derived in this way rather than imposed as a characteristic of a particular functional form.

The formulation reported in (7b) is examined in detail below. To orient discussion we initially make a few background comments. To simplify presentation we re-write (7b) in general form:

\[
g(\varepsilon) = f'(\varepsilon)
\]  
(7c)

Earlier discussion suggested that standard transaction cost analysis of the firm derives a positive relationship between transaction complexity and internalisation. In terms of (7c), and the modelling presented here, this standard perspective requires \(g'(\varepsilon) > 0\).

Defining \(g'(\varepsilon)\):

\[
g'(\varepsilon) = \frac{-\delta \alpha e^{(\varepsilon-1)} + \delta^2 - (\varepsilon - 1)\alpha e^{(\varepsilon-2)} + (2\varepsilon - 1)\alpha e^{(2\varepsilon-2)} - \delta e^2 e^{(\varepsilon-4)}}{2\beta}
\]  
(7d)

It follows from (7d) that positive \(g'(\varepsilon)\) requires a positive numerator. This latter condition can be presented in the following way:

\[
\varepsilon[\delta e^{(\varepsilon+1)} - \alpha e^{\varepsilon} + e^{2\varepsilon}] + \varepsilon^2[\alpha e^{\varepsilon} - 2e^{2\varepsilon} + d e^{(\varepsilon+1)}] - d^2 e^2 < 0
\]  
(7e)
It is clear from (7e) that a sufficient condition for \( g'(e) > 0 \) is \( \varepsilon = 0 \). This follows because \( d^2e^2 \) is strictly positive and hence \( -d^2e^2 \) is negative. A requirement that \( \varepsilon = 0 \) implies that organisational effort and production costs are independent, a feature of traditional transaction cost theory as discussed above. The more general case of non-zero \( \varepsilon \) introduces the polynomial defined in (7b) or (7d). This more general case is most readily explored with particular parameterisations.

In the simulations of (7b) reported below four values for \( \varepsilon \) are used. When effort has minimal affect on average production costs \( \varepsilon \) is assumed to be -0.1 and -0.5. With effort having a significant impact on unit production costs the absolute value of the elasticity is taken to be greater than one at -1.5 and -2. We can view \( \varepsilon = -0.1 \) as close to standard TCE theory and hence we would expect somewhat standard predictions. On the other hand we can view \( \varepsilon = -2 \) as somewhat inconsistent with standard TCE and hence any conclusions that differ from those expected by standard TC analysis of the firm would indicate that analysis of interconnections between institutional and technical features of the firm do indeed matter i.e. there is intellectual value added.

Two values for potential market size are used: \( \alpha = 10 \) and \( 100 \); it is an obvious point that these values of \( \alpha \) should be given an ordinal interpretation. For all results \( \beta = 0.5 \) and \( \delta = 1 \) are used. In addition, the results report effort scaled from zero to 100 and are summarised in the Figure 3a-3h.

Figures 3a-3h here, see end.

In these diagrams the horizontal axes are defined by \( f'(e) \) i.e. an indicator of transaction complexity. The vertical axes show the level of organisational effort that results from any particular \( f'(e) \). First consider figure 3a. Here there is a small potential market and minimal impact of effort on production costs. The result is that we have a threshold level of \( f'(e) \) beyond which organisation effort is rational followed by a positive relationship. This result is consistent with traditional TCE as discussed above in the context of the comparative institutional method that is used. Turning to figure 3b, the only change compared to 3a is a large potential market. Ignoring the non-monotonic relationship (for the moment) we see that increasing \( \alpha \) shifts the relationship to the right i.e. raises the threshold \( f'(e) \) beyond which organisation effort becomes rational. This can be interpreted in the following way: a
larger potential market allows the management of transaction complexity over a wider range without having to internalise activity. In concrete terms this might involve greater use of subcontractors and external distributors and the like in large markets. But while the absolute impact of transaction complexity appears to interact with general market features the marginal impact appears exogenous i.e. the upward sloping parts of the diagrams have constant slope. If we adopt a monopolistic competition interpretation of the single firm model used here we can understand the shift from figure 3b to 3a (i.e. lower \( \alpha \)) in terms of firm entry. In this case firm entry (i.e. greater competition) results in firms being more responsive to contract complexity. This endogeneity of the threshold \( f'(e) \) to \( \alpha \) is apparent in all the diagrams i.e. when \( \alpha \) changes with a constant \( \varepsilon \).

The second feature of the diagrams to be discussed here concerns the impact of increasing the absolute size of \( \varepsilon \) i.e. making production costs more responsive to organisational effort. The relationship between \( f'(e) \) and \( \varepsilon \) is non-monotonic, apart from when \( \alpha \) and \( \varepsilon \) are both small in absolute terms. We can interpret this non-monotonic relationship as follows. Beyond the threshold \( f'(e) \) two organisation solutions exist that define “large” and “small” organisations. Note that earlier discussion of (6a) suggests that all positive effort levels have positive physical output. The small organisation solution has an upper limit: there is some degree of \( f'(e) \) that renders small firm activity unviable. This upper limit depends on both \( \alpha \) and \( \varepsilon \). A larger potential market increases the scope for small scale activity. In addition, a larger impact of effort on unit production costs increases this scope. This result appears to identify a rationale for niche producers and the coexistence of small and large firms even with constant returns in production.

To generalise this discussion of figures 3a to 3h, we see two types of firms:

1. Firms with potentially large organisational size managing increasing contracting complexity by increasing organisation capacity. Discussion in the next section qualifies this conclusion to some extent.

2. Small firms can manage increasing contracting complexity by exploiting advantages of small organisational size, particularly when production cost advantages exist and in large markets. This effect has an upper limit in the diagrams.
If we view industry evolution as occurring from atomistic markets with small firms (as in 2) to markets with characteristically large firms (as in 1), it is clear from the model developed here that this development can occur because of marketing and/or production cost advantages of larger scale organisation; a conclusion not inconsistent with Chandler (1962, 1990). We can view this firm growth and evolution as occurring in one of two ways. First, a successful firm might grow incrementally in “small” steps from small to large size. Secondly, firms might undertake a developmental leap by “jumping” from the small firm solution (in 2) to a large firm solution (in 1). The relevance of these two possibilities is examined in the next section. Finally in this section we point out that if evolution does not reduce the absolute size of $\varepsilon$ we can see the coexistence of small and large firms. To reiterate a point made earlier: this set of results is based on constant returns to scale in production i.e. it is generated by organisational factors and the impact of these factors on technical firm functioning.

**Firm profitability: viability and organisation equilibrium**

Up to this point in the discussion two important issues have been ignored: (1) that any firm must earn non-negative profits for long-run viability; and (2) that the small and large firm effort solutions differ in important respects apart from size. These matters will be considered in turn in this final substantive section. We will see that both (1) and (2) qualify, but do not fundamentally change, the analysis in the previous section. In addition they allow us to draw conclusions about “small step” compared to “developmental leap” strategies.

To analyse viability we use the earlier defined profit function, reproduced here with a non-negativity condition:

$$\pi = \alpha x - \beta x^2 + \delta x e - xa(x,e) - f(e) \geq 0$$  \hspace{1cm} (9)

By assumption TCs (i.e. $f(e)$) are a linear function of effort with zero vertical intercept, hence $f(e) = ef'(e)$. Introducing this into (9):

$$\pi = \alpha x - \beta x^2 + \delta x e - xa(x,e) - ef'(e) \geq 0$$  \hspace{1cm} (9a)

Using (5b) to ensure profit maximising effort:

$$f'(e) = x\delta - xa'_e(x,e)$$  \hspace{1cm} (9b)

Substituting (9b) into (9a) and simplifying, firm viability requires:

$$\alpha - \beta x - a(x,e) + ea'_e(x,e) \geq 0$$  \hspace{1cm} (10)
To compare (10) with earlier results we can use the same specific formulation: \( \beta = 0.5 \), \( \delta = 1 \) and \( a = x^0 e^\epsilon \). Using this formulation and re-arranging (10) implies a constraint on output:

\[
x \leq 2[\alpha + (\epsilon - 1)e^\epsilon]
\]  
(10a)

Using (6b) this output constraint for viability can be re-written as a constraint on transaction complexity i.e. \( f'(e) \):

\[
f'(e) \leq 2[\alpha - \alpha \epsilon^{(\epsilon-1)} + (\epsilon - 1)e^\epsilon - \epsilon(\epsilon - 1)e^{(2\epsilon-1)}]
\]  
(11)

The inequality (11) indicates that firm viability is subject to an upper constraint on transaction complexity i.e. excessive transaction complexity produces negative profitability. To compare the implications of this inequality with earlier results we can use the particular parameter values for figures 3a and 3h with the intermediate cases being somewhat obvious in the light of earlier discussion. Figures 4a and 4b show the results of this exercise: 4a uses \( \alpha = 10 \) and \( \epsilon = -0.1 \), and 4b uses \( \alpha = 100 \) and \( \epsilon = -2 \). Values of \( f'(e) \) to the left of these curves are viable solutions.

Figures 4a and 4b here, see end.

In figure 4a the constraint produces an upper limit on organisational size \( (e) \) i.e. only the bottom segment of the solutions identified in figure 3a is viable. Given the parameter values used in 4a and subject to what is discussed below, this does not change the basic relevance of the conclusions drawn about TCE i.e. increasing transaction complexity increases TCs but subject to an upper limit. Figure 4b should be compared with the solutions identified in figure 3h. Examination of these two diagrams indicates that all the small firm solutions are viable but large firms face a constraint, subject to what is discussed below. Once again, the conclusions drawn are qualified but not subject to fundamental change when viability issues are introduced.

We now turn to the second issue identified at the start of this section: that the small and large firm effort solutions differ in an important respect apart from size. The most straightforward way to examine this matter is to formulate the relationship between effort and profit. Using the profit function defined earlier in (4) and incorporating profit maximising real output, as defined in (6a), along with \( a = x^0 e^\epsilon \):
\[
\pi = \left[ \frac{\alpha - e^e + \delta e}{2\beta} \right] \left[ \alpha - \beta \left( \frac{\alpha - e^e + \delta e}{2\beta} \right) - e^e + \delta e \right] - f(e) \quad (12)
\]

Introducing into (12) the formulations adopted earlier in this section i.e. \( \beta = 0.5, \delta = 1 \) and \( f(e) = ef'(e) : \)

\[
\pi = \left[ \alpha - e^e + e \left( \alpha - 0.5(\alpha - e^e) - e^e + e \right) - ef'(e) \right] \quad (12a)
\]

Formulation (12a) defines firm profits as a function of organisation effort, along with the fundamentals of the model, assuming profit maximising output. We will consider the same two cases as just discussed: (1) \( \alpha = 10 \) and \( \varepsilon = -0.1 \), and (2) \( \alpha = 100 \) and \( \varepsilon = -2 \).

The results for (1) are shown in Figure 5a and for (2) in Figure 5b. In each diagram three different levels of contract complexity are shown; the levels of \( f'(e) \) are necessarily larger with \( \alpha = 100 \), compared to \( \alpha = 10 \), for reasons that follow from earlier discussion.

Consider initially Figure 5a i.e. a small market and minimal impact of effort on production costs. This diagram corresponds to the situations in Figures 4a and 3a. With \( f'(e) = 15 \) it can be seen that the profit function has as internal maximum at small effort (approximately \( e = 0.2 \)) and an internal minimum (but still positive profit) at larger effort (approximately \( e = 6.0 \)). At higher effort than the internal minimum the profit function is monotonic positive i.e. it is possible for the firm to earn greater profits than the internal maximum at some large effort (in the region of \( e = 10.5 \)). It follows that the internal minimum (\( e \approx 6 \)) can be viewed as defining a minimum organisation size beyond which firm growth increases profitability. In terms of earlier discussion the small firm solution is a constrained profit maximum and the large firm solution a minimum organisation size for a large firm. With \( f'(e) = 15 \) it is possible for a small firm to develop using a “small steps” incremental strategy and shift from the small to the large solutions because at no positive effort levels are losses made. There is, of course, a transition or adaptation cost involved here, with a “small steps” strategy, in terms of (temporarily) declining profitability until the large firm minimum size is achieved. This effort-profit curve, with \( f'(e) = 15 \), lies to the left of the viability constraint in Figure 4a hence all positive effort levels are viable.
Now consider the case of greater contract complexity in Figure 5a, i.e. \( f'(e) = 20 \). The small firm internal maximum still exists and is viable but the large firm minimum size is now not viable. In terms of Figure 4a we have shifted to the right of the viability constraint. But there is still a large size that is significantly greater than the internal minimum that generates positive profits. This large organisation size is approximately \( e = 16.25 \) compared to the small firm solution of approximately \( e = 0.2 \) i.e. a small firm must grow by a factor of greater than 81 times its small size to achieve viability. Hence we must qualify the earlier viability discussion. Increasing contracting complexity eventually undermines a “small steps” incremental strategy because the transition-adaptation costs become excessive. In principle a developmental leap may be possible but the extent of the required “jump” may render this non-viable.

Finally, in the context of Figure 5a, consider now the case of reduced contract complexity i.e. \( f'(e) = 10 \). Here the effort-profit curve is monotonic positive at all positive effort levels; in short there is no internal profit maximum at small effort. In terms of the earlier Figure 3a this level of \( f'(e) \) is to the left of the threshold value, hence profit maximising effort for small firms and the minimum effort for large firms both have no positive effort solutions. This undermines constrained small firm functioning (i.e. the same conclusion as drawn earlier) but does not undermine (what is called here) large firm activity that is still viable. In the context of Figure 4a, with \( f'(e) = 10 \) we are to the left of the viability constraint.

This discussion of \( f'(e) = 10 \) in Figure 5a suggests that the small and large effort solutions are not merely different firm sizes but also different firm types. The small effort (internal) solution appears to describe firm existence in a manner consistent with transaction cost theory. At low transaction complexity (i.e. low \( f'(e) \)) small firms are not viable but become so beyond some critical \( f'(e) \). The large organisation minimum effort solution characterises firms based on the inter-linkages between organisation effort and the marketing and production cost effects rather than simply firm existence based on transaction cost factors. In effect the large firm development is closer to a Chandler-type (1990) analysis of the firm. The shift from a small firm (transaction cost) analysis to a large firm (marketing and production cost) analysis depends on environmental complexity that determines the adaptation costs involved. With minimal contract complexity, e.g. with \( f'(e) = 10 \) in Figure 5a, there is a special
case with zero adaptation costs of firm development hence a small steps development strategy is effective. With greater contract complexity the adaptation costs increase until they eliminate the possibility of a small steps strategy and also may make a developmental leap unlikely because of its size. In this case, subject to what is said below, we appear to be left with a transaction costs analysis of (small) firms.

Consider now Figure 5b that is based on a larger market (α=100 compared to 10) and significant impact of effort on production costs (ε=-2 compared to -0.1). This diagram corresponds to the situations in Figures 3h and 4b. An initial point is that all three levels of contract complexity shown in Figure 5b have viable small firms i.e. there is an internal maximum at small effort. But it is clear from Figure 3h that with lower f'(e) small firm activity is undermined and we enter the special-case world of zero firm development adaptation costs that was just considered. This is the special case world in which transaction costs cannot account for firm existence but instead firm development is based on marketing and production cost effects of organisational activity. This suggests that with market growth, or monopolisation, but with no increase in contract complexity, eventually a barrier to small scale activity emerges.

The second point to emphasise with regard to Figure 5b concerns the extent of the adaptation costs at differing contract complexity. An increase in f'(e) from 175 to 200 increases these dynamic costs but does not undermine a “small steps” strategy. But a further increase in f'(e), and greater dynamic adaptation costs, from 200 does undermine such a strategy. Hence significant contract complexity implies that firms must rely on a risky developmental leap strategy.

The analysis presented here suggests that the “real firms” framework has two firm types as particular solutions: (1) the existence of small firms, the analysis of which appears consistent with transaction cost theory; and (2) large marketing, production cost based firms where the effects of organisational effort impacts on real activity. In addition strategies to switch from the first to second firm type depend on environmental complexity. At small f'(e) a small steps strategy is possible but then a developmental leap is required, and finally a shift appears not feasible. But this analysis of development strategies is based on the organisational assumptions discussed above. These assumptions imply not only linear organisation costs but also fixed organisation technology with no organisational innovation. But it is clear from
the organisation development literature, for example the shift from U to M form structures discussed by Chandler (1962) and Williamson (1975) that firm development can involve organisational innovation.

A U form structure can be rationally used by small firms and also by large firms in relatively stable conditions. In terms of the framework developed here, relative stability implies low transaction complexity and hence the possibility of a small steps strategy to shift from small to large solutions. But in complex conditions use of a U form structure in large firms generates non-viability because of control loss, an effect that is predicted by the real firms framework developed here. The implication is that when neither small steps nor developmental leap strategies are viable firm development requires organisational innovation, as with the development of the M form structure that can more effectively manage both size and transaction complexity. In terms of the transaction cost curves depicted above in Figure 1, the use of an M form structure may involve curves with a lower slope, and hence a greater ability to manage complexity, but a positive vertical intercept because of head-office organisational activity that has no direct impact on marketing and production costs. Using this interpretation, diagrams such as Figures 5a and 5b will have two sets of curves corresponding to each organisational technology.

Conclusion
The discussion presented above has been motivated by three key themes, as set out in the introduction: to integrate TCs into a standard model of the firm; to examine the interaction between organisational factors (TCs) and standard demand-cost factors; and to analyse key propositions of transaction cost economics (TCE) with the general model. Given these themes, two sets of conclusions can be reinforced here. First, we can account for the existence of small firms that depends on a threshold value for transaction complexity; an observation that appears consistent with TCE. Secondly, the interaction between organisational and real activity produces a large firm solution. The development of these large firms is based on marketing and production costs management and hence are characterised as more in the tradition of Chandler rather than TCE. Furthermore it is shown that the strategies that can be used to shift from small to large firms depend on transaction complexity. With increasing complexity eventually a small steps strategy is not possible because of the transition costs
involved. In these circumstances firm development requires a developmental leap. This result, of (potential) multiple organisational solutions, is consistent with results from some time ago by Pagano (1992) and Pagano and Rowthorn (1994). While both these latter papers and the approach developed here have grown out of transaction cost theory and have similar conclusions, the theoretical frameworks used are somewhat different. But collectively they can be seen as suggesting an important organisational feature that is often overlooked i.e. the possibility of non-unique solutions.

The second matter that can be briefly considered in this conclusion is the single firm framework upon which the analysis is based. In the text this was interpreted either as a monopoly model or in terms of monopolistic competition. In the latter context, the analysis based on constant returns in production is revealing. The interaction of firm entry and organisational factors generates a movement to a zero profit long-run equilibrium i.e. when the viability constraint binds. Reduced market size increases the impact of transaction complexity on organisational size. This reinforces the analytical logic that starts with the impact of organisational conditions on organisational effort and then in turn effort determines physical output. This is consistent with the general approach to the firm that organisation matters and is the key driver in firm development. But an obvious possible development here, beyond the scope of the existing discussion, is to base analysis on other market structures. Obvious possibilities here are: to use a Cournot-type oligopoly context, or analysis of different output pricing policies, or different market entry conditions and the differing impacts on organisational solutions. Hence incorporating the framework developed here into alternative contexts offers fruitful areas for future work.
References
Chandler AD (1990), Scale and Scope: the dynamics of industrial capitalism, Cambridge: Harvard University Press.
Figures

Figure 1: Linear organisational technology and transaction complexity.

![Figure 1: Linear organisational technology and transaction complexity.](image)

Figure 2: The effort-output relationship.

![Figure 2: The effort-output relationship.](image)
Figure 3a

$\alpha = 10$, $\varepsilon = 0.1$

Figure 3b

$\alpha = 100$, $\varepsilon = 0.1$

Figure 3c

$\alpha = 10$, $\varepsilon = 0.5$

Figure 3d

$\alpha = 100$, $\varepsilon = 0.5$

Figure 3e

$\alpha = 10$, $\varepsilon = 1.5$

Figure 3f

$\alpha = 100$, $\varepsilon = 1.5$

Figure 3g

$\alpha = 10$, $\varepsilon = 2$

Figure 3h

$\alpha = 100$, $\varepsilon = 2$
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