

BACHARACH'S 'VARIABLE FRAME THEORY': A LEGACY FROM SCHELLING'S ISSUE IN THE *REFINEMENT PROGRAM*?

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LAUREN LARROUY

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Bacharach's 'Variable Frame Theory': a legacy from Schelling's issue in the *Refinement Program*?

Lauren Larrouy¹

1. Introduction

In the recent history of game theory, the *refinement program* is often acknowledged to be one of the richest periods from a theoretical point of view (see Giocoli 2003 among others). In this paper, I will compare the contributions of two original game theorists, Schelling and Bacharach, with respects to the *Refinement Program*'s stakes.

The profound debates the *refinement program* has generated constitute contemporary game theory. Besides, it corresponds to the period of the progressive introduction of game theory in economics. In fact, game theory emerged in the 1940s through von Neumann's and Morgenstern's famous *Theory of Games and Economic Behavior* (1944), and Nash's work later in the 1950s²; however, game theory was not as much a success as it was expected. It became a real part of economics only in the late 1970s and 1980s (Giocoli, 2003: 345). To a certain degree, the *refinement program* – which arose in the 1970s –, contributed to a form of enthusiasm towards game theory in economics. This research program generated methodological debates about the foundations of game theory. The fact is that game theory was considered to be too mathematized and too formalized, *i.e.* too abstract, without any “possible applications to concrete socio-economic problems” (*ibid.*, 348).

Schelling's 'vision' of game theory was in complete contradiction with the standard one, which considers – and still consider – game theory as a “mathematical toolbox for economics” (Giocoli, 2003). He attempted to anchor game theory in the real world by highlighting the importance of social events, institutions, conventions, etc. in strategic interaction, to investigate how players could play games. This was a noticeably unusual endeavor to refine game theory. It explains why his work and particularly his book *The Strategy of Conflict* (1960) are still a source of insights in contemporary theoretical developments. Moreover, this book introduced a major concept in Schelling's works which is, to some extent, the cornerstone of his account on game theory: *focal points*. A substantial recent literature attempted to formalize Schelling's intuitions about focal points in a game theoretical framework. However, it is my opinion that, by doing so, Schelling's 'vision' of

¹ University of Nice Sophia-Antipolis – GREDEG/CNRS – Lauren.Larrouy@gredeg.cnrs.fr

² More precisely, Nash's “Equilibrium point in n-persons games” in 1950.

game theory has been garbled. This explains why I prefer to emphasize here the methodological dimensions of *focal points*.

Bacharach, a less known author who was initially interested in input output, matrix analysis, turned quickly to decision theory and to game theory, and tried throughout his entire career and up until his death, to avoid traditional game theory's pitfalls, by investigating (i) the *epistemic assumptions* in games, (ii) the consequences of an *idealized* conception of players' reasoning, and (iii) lastly, *individualism* in game theory.

Both Schelling and Bacharach shared an apparently identical assessment of game theory's limitations, and both attempted to surpass them.

The rationality of players in standard game theory is generally assimilated to what is called *individualism*³, which entails perfect instrumental and cognitive rationality. I will concentrate on Schelling and Bacharach's attempts to escape from these constraints. Indeed, I will argue that one of the main questions addressed by Schelling concerns *individualism* within strategic interaction⁴, and that one way to escape from traditional game theory's limitations is to avoid imposing individualist modes of reasoning in games⁵ – as they are still mostly formalized. This challenge will be contextualized in the main objectives of the *refinement program* in order to highlight the innovativeness of Schelling's contribution for the period, and then compare it with Bacharach's later contribution. The way according to which they chose to *refine* classical game theory explains to some extent how they conceptualized focal points, and, by implication, players' rationality.

In the first part of this paper, I will expose the challenges of the *refinement program* with an emphasis on the limitations of classical game theory. The second part will be dedicated to the presentation of Schelling's vision of game theory. I will highlight his conceptions of standard game theory, focal points, and players' mode of reasoning. In the third part, I will focus on a specific part of Bacharach's contribution in the refinement program: the Variable Frame Theory (VFT). In a comparative fashion, I will expose Bacharach's vision of standard game theory, focal points, and players' mode of reasoning. And, finally in the fourth part of the paper I will investigate how they both tried to escape from *individualism*, and assess whether or not they respectively succeeded in doing so.

³ *Individualism* entails that each players tries to maximize is *own* expected payoff; he is “self-centered self-interest” (Sen, 2007).

⁴In strategic interactions, each player has to form beliefs about others' choice and beliefs. The outcome of the interaction depends on the combination of each player's action.

⁵ A game is, according to Bacharach (1976:2) “an activity of two or more human beings whose outcome matters to each of them, and depends in a prescribed way on the actions of all of them.”

2. ‘Classical game theory’ pitfalls and the *Refinement Program*

Beginning with a short presentation of the basic assumptions of ‘classical game theory’, I will highlight the pitfalls that this version of game theory had to face. Subsequently, I will introduce the *refinement program* – the purpose of which was to solve these pitfalls.

First of all, the so-called ‘classical game theory’ corresponds, according to Sugden (2001:115), to the game theory of the late 1980s, and to the way this set of analytical tools is applied to economics. Basically, it is based on perfect rationality and on common knowledge (of the game’s structure and of rationality) (*ibidem*). Indeed, Bacharach (2001:1) accurately added that “classically game theory seeks to derive players’ choices from the theory of rational individual choice under uncertainty, as found in Von Neumann and Morgenstern (1944), helping itself to a certain fundamental solution concept, Nash Equilibrium.”

Thus three mainstays are visible in classical game theory: (i) perfect rationality in the sense of the expected utility theory (EUT), (ii) common knowledge, and (iii) the ‘solution concept’. For the first premise we can deduce that what prevails in game theory is methodological *individualism*. Perfectly rational, players try to maximize their expected utilities in a context of uncertainty, caused by interaction, and, *a fortiori*, by strategic interaction. Consequently each player takes into account all the relevant aspects of the strategic context he is facing. In particular, it is currently admitted that all the relevant information for players is contained in the games matrix. This matrix specifies who the players are, how many they are, their different potential strategies, and the outcome associated with each combination of strategies for each player – *i.e.*, each player’s payoff function. However, in the context of this strategic interaction each player has to form beliefs: (i) on the eventual choice made by other players, and (ii) on other players’ beliefs about their own choice and beliefs, etc. For the second premise we can deduce that: on one hand the structure of the game, *i.e.*, the matrix of the games, and on the other hand the players’ rationality, are of common knowledge. Besides, each player possesses the same rationality. They are all perfectly rational in the sense of the EUT. Therefore, according to Bicchieri (1993), because of this strategic context games require two kinds of rationality: ‘practical rationality’, *i.e.*, instrumental rationality, and ‘epistemic rationality’. The epistemic rationality entails that players have consistent beliefs and further true beliefs (Bicchieri, 1993:55).

Both of these main assumptions of *classical game theory* involve that any ‘rational deduction’ concerning games may be of common knowledge too. This is what Bacharach (1987) called: “the transparency of reasons”. Basically, if a strategy is considered by a player as the best response to the best response of the other players, each player *truly* anticipates this best replying behavior and chooses in this way. As a result, each player acts in this way. This constitutes *the* solution of the game. It leads to third assumption: the ‘solution concept’⁶. In this context the central piece of *classical game theory* was the Nash equilibrium. The implicit

⁶“A *solution concept* is a rule which applies to all games in some general class and which, for each such game, picks out one or more combinations of strategies as *the* solution or solutions.” (Sugden, 2001: 115).

assumption was that, if a Nash equilibrium exists, players are naturally led to attain it. Another implicit and widespread idea related to the 'solution concept', which was formally developed by Bacharach (1987), was that the existence of a solution implies the attainment of this solution.

Therefore, the idea that each player chooses his best response to the best response of the other is normally entirely deduced from the premise that players entail true beliefs, that a consistency of beliefs prevails, that the game's structure is of common knowledge and that each player possesses the same rationality and that this fact too is also of common knowledge. However, in a number of situations these *epistemic assumptions* are insufficient. In games in which there are no strictly dominant strategies, this way of resolving a game is problematical (the attainment of *the* game's equilibrium cannot be sustained). And the fact is that, by the application of game theory to economic cases in the 1980s, situations in which there are multiple equilibria are more numerous than situations in which there is one unique solution (Sugden and Gold, in Bacharach, 2006: xv). In fact, for this kind of situation, Nash equilibrium has to be refined in order to identify one equilibrium as "the rational way to play" (Bicchieri, 1993: 64).

Consequently, in the late 1970s and in the 1980s, game theorists recurrently faced problems related to the "internal consistency" of classical game theory's assumptions and "the indeterminacy of its predictions" concerning players' behavior (*ibid.*, xiii). They faced: (i) the problem of equilibrium selection which poses the question of how to select one of them and why; and (ii) the problem of rationality and what a rational player ought to do in games. It explains why "in the 1980s, finding coherent foundations for game theory was seen as one of the most important theoretical projects in economics." (*ibid.*, xiv).

The refinement program arose in this context in the 1970s and the 1980s. It began exactly as explained by Giocoli (2003: 338)⁷, with two contributions from Selten in 1965 and 1975, but was fully developed in the 1980s. The main aim of this program was to *refine* the requirement needed to attain a unique Nash equilibrium in games⁸. Among the large amount of refinement propositions⁹, three types of theories can be identified according to Bacharach (2006)¹⁰: (i) the '*respecification theories*', (ii) the '*bounded rationality theories*' and (iii) the '*revisionary theories*'. The *respecification theories* attempted to translate a coordination game into another type of game insuring the obtention of the optimal equilibrium without any risk or uncertainty. The '*bounded rationality theories*' relaxed the rationality assumption – and furthermore the perfect rationality. Obviously, the *Evolutionary Game Theory*, belong to this

⁷I refer to his end note in the fifth chapter on page 338.

⁸Both Giocoli and Bacharach agree on this fact. According to Bacharach (2006: 45), the objective of the refinement program was to "eliminate equilibria which were intuitively suspect, by adding to the Nash solution principle further rationality conditions on players' choices of a decision-theoretic character". And according to Giocoli (2003: 322) its objective was to "strengthen[ing] the requirement for NE [Nash Equilibrium] in order to exclude all the equilibria that were in a specified sense, 'implausible'."

⁹Hundreds of concepts have been proposed (*ibidem*)

¹⁰Sugden too in 2001 asserted that there exist three types of *refinements* which are rather different from those of Bacharach. However, presently I have chosen to keep Bacharach's classification. It fits with the way I see Schelling's inscription in the *refinement program*.

category of theories. And finally, the ‘*revisionary theories*’ imply a revision of the notion of rationality; they use different concepts of rationality.

Bacharach (2006) argued that Schelling’s conception of game theory could be considered to belong to these *revisionary theories*. That means that his vision of game theory was based on a new conception of players’ rationality. Indeed, I must emphasize that in Schelling’s theory of focal points¹¹, from different cultural backgrounds, different practical modes of reasoning can be sustained. He used social phenomena, like conventions or the existence of a *common background*, to *revise* classical game theory, inducing new insights into rationality. This was quite revolutionary. And, in fact, lots of studies introducing bounded rationality in game theory arose (we must keep in mind that the refinement program dates back to the 1980s). Very few game theorists were led to investigate the social and collective dimensions induced by the context of a strategic interaction, implying two or more players. However as was highlighted by Bacharach and Hurley (1991: 3), “a number of questions arise about the relationship between individual rationality and game-theoretic rationality” and about “whether games may be embedded within supposedly individual decision problem.”. Even though, this kind of criticism was not really followed, with the exceptions of Schelling (1960) and Gauthier (1975), after them by Hurley (1989), Bacharach (1999, 2001a, 2001b, 2006) and Sugden (1993, 1995, 2000, 2005, 2007).

As it is now acknowledged, the major part of contributions led to a negative conclusion in the late 1980s – all the new efforts to justify the validity of the Nash equilibrium as a solution were not successful; the refinement program had failed (Sugden and Gold, in Bacharach [2006]; Giocoli [2003]). This helps explain why, according to Sugden and Gold (2006: xv), “[Game] theorists turned their attention to evolutionary and behavioral models”. However, it should be emphasized that this may be due to the fact that Schelling’s new insights in game theory were not taken seriously. Taking into account that players and game theorists do differ, and consequently that they cannot share a common practical reasoning, was one of the possible solutions to escape from classical game theory’s pitfalls. Including the players’ mode of reasoning in their individual contexts is a part of this conception; and indeed both Schelling and Bacharach have this in common. In addition, they both advocate the opening of game theory to the other social sciences like sociology, for example.

3. Schelling and his ‘reorientation of game theory’

In this part devoted to Schelling’s insights into the *refinement program*; we will focus on three aspects of his ‘reorientation of game theory’¹² in *The Strategy of Conflict*. More precisely, we will tackle (i) his conception of game theory, (ii) his principle of *focal points*,

¹¹Asserting that Schelling created a *theory* of focal points can be questionable. For this argument we must refer to Sugden and Zamarron (2006).

¹²Schelling ([1960] 1980: 83) explicitly argued by choosing as title ‘a reorientation of game theory’ that his view of games and generally of game theory may participate to a renew of game theory.

i.e., the cornerstone of this *reorientation*¹³, and consequently (iii) the kind of mode of reasoning which is entailed in this conception of game theory – *i.e.*, the type of rationality induced.

Schelling's conception of game theory was highly empirical. His main critic against classical game theory was to abstract games from all empirical facts and from all contextual information, even though they are highly relevant and valuable to resolve games. In this manner, game theorists enable players to resolve coordination games, cooperation games, bargaining games, etc.¹⁴. This fact is obvious in the following critic: "Labeling of *strategies* [...] is implicitly precluded by dealing only with games in normal form, that is, the abstract version of them as represented by a payoff matrix [...]. It is precisely because strategies are labeled in some sense – that is, have symbolic or connotative characteristics that transcend the mathematical structure of the game – that players can rise above sheer chance and "win" these games" (Schelling, [1960] 1980: 95-96). Furthermore, he asserted that "suggestive details" can be interpreted as "clues" or "signal" (*ibid.*, 107-108); they have an influence on the outcome. In fact, the way players play a game reveals information about their intentions and objectives. So the context of play is of interest. It justifies why strategies should not be labeled symmetrically, and why mathematics is not the most cut out for describing interaction situations. Basically, everything is a source of information to players in order for them to resolve games (or strategic situations). Game theorists by expunging games from all kinds of non-mathematical labels induce a real impossibility to solve them¹⁵. Indeed, in Schelling's account, "reactions are subject to a mutual interaction that results from the fact that each can see the other's reaction and each knows that his own visible reaction is yielding information about his own expectations." (*ibid.*, 110). The way players play games provides information to these players. The outcome reached is influenced by this kind of information (which is not deduced by the mathematical structure of the game).

Games' mathematical properties can be relevant only for mathematicians and for game theorists. "If two players are themselves mathematical game theorists, they may mutually perceive and be powerfully affected by potential solutions that have compelling mathematical properties. Each may transcend, and know that the other will transcend, various adventitious details that, to non-mathematician game players, might be more relevant to the focusing of expectations than some of the quantitative properties of the game." (*ibid.*, 113-114). However, according to Schelling (*ibidem*) "We must avoid assuming that everything the analyst can perceive is perceived by the participants in a game, or that whatever exerts power of suggestion on the analyst does so on the participant in a game. In particular, game characteristics that are relevant to sophisticated mathematical solutions (except when that

¹³"the theory of focal points is clearly intended as a principal component of this reorientation [game theory]" (Sugden, Zamarròn, 2006:610)

¹⁴According to Schelling ([1960] 1980), coordination games, cooperation games, bargaining games, etc. are not totally different. They all imply common interests.

¹⁵This is perhaps not a logical impossibility. However, I think that Schelling did not take a position on the logical impossibility of game solving, contrary to all researchers which took part in the *refinement program*.

same solution can also be reached by an alternative, less sophisticated route) might not have this power of focusing expectations and influencing the outcome. They might have it only if the players perceived each other to be mathematicians.”. Consequently, different cultural backgrounds (in this context being either a mathematician or a non-mathematician) imply different modes of practical reasoning.

Another innovative aspect of Schelling’s conception of game theory, which is obvious in his *focal points* principle, is that the traditional categories of games should not be distinguished. In fact, coordination games (*i.e.*, non-zero-sum games), bargaining games and divergent interest games (*i.e.*, zero-sum games) are all games of common interest. All of these types of games require “some kind of collaboration or mutual accommodation” (*ibid.*, 83). However, the prevalent opinion in the game theorists’ community is that coordination games are at the opposite extreme of divergent interest games. This is due to the riskiness for players to coordinate¹⁶. It can be asserted by this originality that, to some extent, Schelling was implicitly confessing that he did not accept *individualism*. Basically, if a player is rational in the sense of the EUT, he does not cooperate or coordinate with the other(s), because it is too risky. By acting like this, he does not maximize his expected utility. We face three options: (i) the player is irrational – it is not the option grasped by Schelling because as such the player avoids a ‘mutual disaster’; (ii) he is not individualistic; and (iii) his behavior is driven by a different form of rationality or mode of reasoning. The following insights into the Schelling theory will give us some clues to argue that Schelling retained the third option.

It is now important to look at Schelling's central notion: the *focal points*. Focal points are induced by situations in which “two or more parties have identical interests and face the problem of [...] coordinating their actions for their mutual benefit, when communication is impossible.” (*ibid.*, 54). Consequently this principle can be applied indifferently to coordination games, cooperation games and divergent interest games. In all these types of games “[players] must “mutually recognize” some unique signal that coordinates their expectations of each other.” (*ibidem*). And, the fact is that “Most situations – perhaps every situation for people who are practiced at this kind of game [that is, common interest game] – provide some clue for coordinating behavior, some focal point” (*ibid.*, 57). In fact, some solutions are “qualitatively different” (*ibid.*, 70) from the other, in the set of all the possible solutions. These types of solutions are prominent. And finally Schelling argued that “it is a prominence that depends on time place and who the people are.” (*ibid.*, 57-58). This fact is highly important because it confirms, again, that for Schelling's game theory (in this context via the focal points) should be embedded in social dimensions, in the players’ context for the games, etc. Focal points exist only because of these ‘clues’ which occur in the real world. And the fact is that in the real world, players interact and coordinate every day. These clues can be perceived by players only if they pay attention to the context of their interactions. In this context, players acknowledge that they share a common capacity, to find and to understand,

¹⁶This opinion was defended by Bacharach. Cooperative games are too risky for an individualist player to cooperate.

these clues. If players are embedded in different contexts, and face an identical problem of coordination, in each context, the way they resolve this problem and the solution will be different. In this manner, we can argue that he has *respecified* the players' way of reasoning in games.

However, this kind of reasoning is still based, in order for players to resolve the game, on convergent expectations. "The outcome is determined by the expectations that each player forms of how the other will play, where each of them knows that their expectations are substantially reciprocal. The players must jointly discover and mutually acquiesce in an outcome or in a mode of play that makes the outcome determinate. They must together find "rules of the game" or together "suffer the consequences." (*ibid.*, 107). In this quotation, it seems that there is no need for an infinite regression of beliefs as is currently the case in classical game theory. It gives the impression that resolving a game is a collective process; together players will discover the solution which will induce convergent expectations. Using his example of a couple, a husband and wife, who are lost in a supermarket, again coordination can be interpreted as a collective enterprise.¹⁷ However, the fact is that they are able to find each other, *i.e.*, to resolve this coordination problem (each of them has to go in the same place, so we face here a pure coordination game) by thinking of a place that is so obvious that each of them knows and is 'sure' (term employed by Schelling [*ibid.*, 54]) that this place is obvious for each of them. In this example and in the following quotation this problem of coordination is resolved by the faculty that each member of the couple reacts as if he/she is in the other's shoes. There is an intersubjective faculty in each member of the couple's thoughts. Furthermore, even more than this intersubjective capacity, each of them knows the other. Thus, they can be considered (via their mutual knowledge) as a kind of *collective entity*, even if this doesn't imply collective agreement and collective action in the strong and restrictive sense of Gilbert (1989, 2000, 2003). Let us consider a member of this collective entity. By his knowledge of the other member, he knows how they will be able *together* to find a solution. This is why, in my view, they form a group with a distinct mode of reasoning. Each member of the group use the fact that, putted together, they become an entity with a common fate. Each player uses his knowledge about the other in a way that gives information to him about the kind practical reasoning they will use together. Consequently, the resolution of this coordination problem is due to each person's knowledge about the other *and* each person's knowledge of the context. In fact, players have "to read the same message in the common situation" to solve the game, that is "to identify the one course of action that their expectations of each other can converge on." (Schelling, [1960] 1980: 54). And, according to Schelling (*ibid.*, 84), in common interest games "Players have to understand each other, to discover patterns of individual behavior that make each player's actions predictable

¹⁷Sugden and Zamarròn (2006) point out that the example of the couple is question-begging. But finally they assert that it is not a collective agency as Gilbert (1989, 2000, 2003) defines it: *i.e.*, for example, two or more people who decide together, by a common agreement, to accomplish together a collective and interdependent action. It is true that if we retain this definition of a collective agency, it is too restrictive and cannot be applied to Schelling's example.

to the other; they have to test each other for a shared sense of pattern or regularity and to exploit cliché conventions, and impromptu codes for signaling their intentions and responding to each other's signals.”. There is reason to believe, again, that some social dimensions can induce one ‘pattern of behavior’ which becomes qualitatively better than the other, according to the context, and implying convergent expectations. More than players’ mutual knowledge about themselves, it is because players are socially embedded that a solution emerges.

4. Bacharach’s Variable Frame Theory

I will now analyze Bacharach’s work, especially his Variable Frame Theory (VFT) which led him to formalize new types of games: the Variable Universe Games (VUG), in light of Schelling’s precedent insights into classical game theory. I will follow the same sequential schema as for Schelling. After a short introduction of Bacharach’s VFT I will investigate Bacharach’s vision (i) of game theory, (ii) of the focal points and finally (iii) of players’ rationality and mode of reasoning. Two components of his theorization of game theory: (i) epistemic foundations of games and players’ rationality, and (ii) players’ frame are effective in his VFT. Thus, I will attempt to explain this VFT – an original and apparently difficult.

Like Schelling, Bacharach asserted that: “conventional game theory confuses the world as seen by the theorist with the world as seen by the decision-making agent [...] it is essential to distinguish between the decision that appears in the theorist’s model and the problem as the agent represents it to herself.” (Bacharach, 2006, xvi). This approach is totally integrated in the VFT. By taking into account players’ frames, traditional games (i.e., games in a normal form (the matrix) or in an extensive form) are changed in what Bacharach called VUG. The VUG “specifies *the way players conceive their situation* and how this varies” (Bacharach, 1993: 256).

Three main premises entail the formalization of the VFT: (i) players must present themselves with a set of alternatives (i.e., the subjective strategies) among which they have to choose, (ii) a model must specify what the players’ frames are, and (iii) finally it must involve principles of *equilibrium selection* and of what games’ equilibrium is (Bacharach, Bernasconi, 1997:3). Therefore, a game in his VFT is deduced from the standard games as they are specified by game theorists. Basically from a set of action and a set of outcomes (there is a given outcome for each combination of action), another game is built. To each *objective* feasible act or strategy, i.e. not framed by the players, an act description is associated. For each player there is an ‘*act-description space*’. In fact, “what option a player has depends on what concepts she has is responsible. It is this which accounts for the payoff matrices being the 1×1 and 3×3 matrices [...] rather than the 5×5 matrix. ”; “It depends on what concepts are available” (*ibid.*, 267).

To build his VFT, Bacharach (in Bacharach, 1993; Bacharach and Bernasconi, 1997; Bacharach and Stahl, 2000; Bacharach, 2001) used a labeling game¹⁸. The principle is still the same when players face a formal coordination game or a formal cooperation game; from objective strategies and outcomes, as the game theorists define them, they frame ‘subjective alternatives’ from which they are able to coordinate. However, as labeling games seem more intuitive to explain VFT principles, I will present here the changes that occur through the integration of players’ frames in labeling games.

Bacharach assumed that for all physical objects there is a set of concepts which belong to a set of ‘families’. Let us use an example to understand Bacharach’s concepts; and precisely a red balloon. A balloon can be described by the ‘*color family*’ – or equivalently ‘colors concepts’. The balloon can be described by other concepts like ‘shape concepts’, ‘aspect concepts’, ‘material concepts’, etc. A set of families or concepts can be applied to a physical object and for Bacharach (1993: 260) it corresponds to a ‘repertoire’. When particular *attributes*¹⁹ – basically, features or characteristics for the object – which belong to a given family, come to a player’s mind, these attributes which all belong to the given family come in a ‘cluster’. An ‘*availability function*’ defines the probability that a player possesses a certain repertoire²⁰ and *a fortiori* a certain set of families. This probability is independent from one player to another. Indeed, players’ frames correspond to the repertoire they dispose; or, in other words, to “*the set of families that come to her mind*” (Bacharach, Bernasconi, 1997:6). Players’ frames can be more or less complex, depending (i) on the number of families – or concepts – used to describe the object and (ii) the associated probabilities for each family to come to the players’ mind. Yet, there is a ‘*coverage function*’ which symbolizes the fact that ‘act descriptions’ occur only for descriptions that belong to the available families. In fact, a “player’s option must be descriptions of possible actions which only use attributes *in her frame*.” (*ibid.*, 6). There is only one option for each ‘*attribute*’ in each family of the player’s repertoire. And, to go one step further, there are only two possible options – depending on the uniqueness of the object or of the characteristic of the object: it is (i) ‘choose at random’ or (ii) ‘pick it’/‘take it’ (never mind). For the balloon if the repertoire of the player is composed by two families: color and shape; the color family is composed by {red, yellow, blue, violet} and the shape family is composed by {oval, round}; the set of act description is {choose the red, choose the yellow, choose the blue, choose the violet, choose the round, choose the oval}. So the player’s available actions are defined in the set of these possible actions. Thus, the player’s *belief-space* about possible actions is restrained by this set. Explicitly, if the set of

¹⁸A labeling game is a game in which players face a set of different objects that can be identified by their color, shape, etc.; and the objective for the players is to choose the same object in order to win money (if they don’t, they fail to coordinate and win nothing).

¹⁹This word was employed by Bacharach and Bernasconi in 1997 but not in Bacharach’s previous work on this topic, even though this idea was already present. In the ‘Color concept’, yellow, red, blue, etc. are attributes; and in the ‘shape concept’, round, square, oval, triangular, etc. are attributes. The fact is that if a person can describe an object by saying that this object is red, the cluster of attributes red, yellow, blue, etc. come to her mind because she is able to distinguish red from yellow, blue, green, and so on.

²⁰In fact, a ‘repertoire’ in Bacharach’s account corresponds to a set of families.

options that a player has, is {choose the red, choose the yellow, choose the blue, choose the violet, choose the round, choose the oval}, it defines his *belief-space*. The player cannot imagine that other options might eventually be available so he is not aware of it. The same is true for his beliefs about the other players' available actions, as well. We can say the same for the utility; a player believes that he can reach a certain payoff according to the combination of his action and of the others' actions. But the possible act for the other players must belong to his own set of available acts. So the repertoire of each player must be a subset of the others' repertoires. Bacharach supposed that each player believes that the others' repertoires cannot be different from his own; it can be a subset or eventually the same but not another one. It is intuitive because we can imagine that another person holds a concept of which we are not aware. Bacharach (1993: 261) assumed explicitly this 'naïve' assumption about players' reasoning. In the example of the red balloon if the player possesses the set of act descriptions {choose the red, choose the yellow, choose the blue, choose the violet, choose the round, choose the oval}, he can only believe that at best the others' set of act descriptions are the same, or a subset. Consequently the payoff for each player depends on each player's *belief-space* and sets of actions. "Rational players must therefore have beliefs about the other player's frames." (Bacharach, Stahl, 2000: 224). And, in fact, "Rational VFT imposes a restriction on these beliefs: the no refinement principle (NR) that the belief spaces considered possible for another player cannot refine a player's belief space." (*ibidem*).

A few mechanisms of equilibrium selection exist. More precisely, two basic principles are involved: (i) rarity and (ii) availability. The solution of a labeling game will be to mark the rare object if the probability assigned by a player to the fact that the other player perceives the rare characteristic or object exceeds the probability of success by choosing at random another object which is not rare ("*Ceteris paribus, players prefer to pick an attribute that is rarer.*" [Bacharach, Bernasconi, 1997: 11]). Rarity and availability can compete (like in 'tradeoff games'). In fact, there is a preference named the 'availability preference' which stipulates that "*Ceteris paribus, a player is more inclined to pick an attribute which is more available*" (*ibid.*, 10). In fact, in the case of tradeoff games 'obviousness' and 'uniqueness' which are, according to Lewis (1969), the two features required for a solution to be a focal point, are not possessed by the same object. Two more principles of selection for a VF-equilibrium were cited by Bacharach and Bernasconi (1997): the first one concerns the *payoff dominance*²¹, and the second one concerns the principle of *symmetry disqualification*. The former stipulates that *ceteris paribus* players prefer to choose 'salient options', that is – in Bacharach's account of VFT – unique options (which, in his analysis, are associated with less risky choices, which insure that players will maximize their expected utility). Precisely, the *payoff dominance* principle is deduced by choices which have the higher probability to be chosen by all players of the game. The latter stipulates that "*if two options a and a' are alike in all relevant respects, a solution concept should not pick out one rather than the other.*" (*ibid.*, 11). Equal options cannot be picked; their associated attribute should be of a different availability to be

²¹ If an outcome entails a larger amount of payoff compared to the other players, if they are instrumentally rational and if they know that they are both rational (in the case of a two player game), they will choose to reach this outcome. But the fact is that in Bacharach's VFT, the explanation of the payoff dominance is different. This payoff dominance is explained by a non-risky choice.

chosen. In the example of the red balloon, if this red balloon is mixed with other balloons and if we add three more balloons, now two of them are red, and two of them are yellow. Players cannot choose at random one of the red balloons or choose at random one of the yellow balloons. Each one of these actions has the same probability.

I must now look at the principle of the focal point, to see how this principle appears and is formalized in Bacharach's VFT. Due to his VFT, Bacharach was able to justify the validity of the focal points, for game theory. He argued that "Although the idea of a focal point seems intuitive, game theorists had never succeeded in explaining precisely what a focal point is, or why it is rational for each player to choose her component of the combination of strategies that constitute one." (Bacharach, 2006, xvi). To the contrary he estimated that he had succeeded in doing it. In this manner Bacharach claimed that he was able to explain how a Pareto dominant equilibrium is *the* solution, he had "only to invoke the so called Principle of Coordination (the principle that in pure coordination game with multiple equilibria, it is rational to play one's part in the unique Pareto-optimal equilibrium if there is one)." (Bacharach, 1993: 257) in the VUG. In the VFT, this 'Pareto-optimal equilibrium' is the salient option, i.e., the focal point. This principle, according to Bacharach, is deduced by '*common sense*' even if this principle is applied to ideal forms of interaction or reasoning. Therefore, contrary to Schelling, it is the formal and mathematical characteristic of the game which leads to a solution; it is not from "payoff-irrelevant features" (Heat et al., 2006: 637) that coordination occurs. Furthermore, VFT transforms pure coordination games in Hi-Lo games – that is, games which have a Pareto dominant strategy and a Pareto dominated strategy. It implies that in coordination games the solutions for each player are to "choose the salient options" (Bacharach, 1993: 256), and *a fortiori* to choose the Pareto dominant strategy. More precisely players frame the objective strategies, defined by the game theorists, by {give the highest payoff; give the lowest payoff}. If {give the highest payoff} is recognized as the salient option, then the focal point is {choose highest payoff}.

Some intuition in Bacharach's account of the focal point principle seemed to be inherited from Schelling's, like the fact that "saliency is shared" (Bacharach and Bernasconi, 1997: 7). Therefore, to some extent it can be assumed that they recognize the existence of a common background – or a cultural background – shared by members of a same community culture, etc. like Schelling. This fact leads to a mutual recognition of this salience and to the fact that each knows that the other knows, etc. Consequently it is enough to assume a primary and a secondary salience²². Bacharach and Bernasconi (1997:7) also asserted that "A player is assumed to have correct beliefs about availabilities for others". This is due, to some extent, to the players' "ability to solve coordination problems such as rendezvous by collectively alighting on a salient equilibrium or "focal point"". (Bacharach, Stahl, 2000: 224). Even so, I affirm that the principle by which an option becomes the solution in Bacharach's VFT is

²²For these terms, we must refer to Metha, Starmer and Sugden (1994). The primary salience is related to the availability of an option. The secondary salience is linked to the availability asserted by a player to the other players' option availability. The latter is linked to players' beliefs.

totally different. Even if Bacharach acknowledged that, to some extent, a common background can justify the sharing of an option, in practice he did not use this fact as a premise to build on his theory. An option can be salient for one of the players, but if this option is too risky because the probability of availability for this option assigned for the other players is too low, then choosing this option will not be the solution. Moreover, the repartition function of the availability probabilities is provided exogeneously.

Finally, by creating the VFT, Bacharach's aim was to consider the players' valid mode of reasoning in taking into account their cognitive constraint. By doing so he attempted to overpass the pitfall into which classical game theory tended to fall. His objective was to "reconstruct game theory with a better model of reasoning – not so much a more realistic model [...] a truer representation of what valid reasoning is." (Bacharach, 2006: xvi). Notwithstanding, Bacharach continued to model rational players. In fact, being rational for him is reaching the Pareto optimality; basically being rational is only having an *instrumental reasoning* and reaching the best solution, nothing more. By making reference to this quotation we understand the distance which exists from Schelling's assertion on players' reasoning. Besides, Bacharach was still confined in some aspects, to traditional game theory rationality. I will treat this point more in detail in the next part of this analysis.

5. Comparing Schelling's and Bacharach's views of game theory, focal point and rationality.

Even if some similarities can be found between certain aspects of Schelling's and Bacharach's respective modes of theorizing game theory, focal points, and perceiving rationality substantial divergences remain.

First of all Schelling's ambition was to bypass classical game theory and the classical way of formalizing players' rationality, while by contrast, Bacharach's ambition was to maintain his approach anchored in traditional game theory. Bacharach only attempted to enrich and widen traditional game theory. In fact, he asserted (in 2006: 58) that his VFT should not be considered as a revolution in classical game theory but as a simple extension of it. In addition, Sugden and Zamarròn (2006:611) argued that "Schelling's discussion of focal points is not a theory as economists normally understand the term. There is no shortage of models, but these are not self-contained worlds in which agents act according to pre-specified principles. They are open-ended coordination problems, designed to be faced by two or more real (as opposed to model) individuals." To the contrary, Bacharach proposed a more or less self-contained model. He used a 'pre-specified principle of reasoning'. Players maximize their expected utility and entail beliefs (true beliefs) about others' beliefs and choices, etc. It is obvious, when Bacharach highlights the fact in his publication (Bacharach, Stahl, 2000:224) that "Rational VFT imposes a restriction on [these] beliefs".

Therefore, Gold and Sugden (in Bacharach, 2006: 27) may be right when they argue that "Bacharach's and Schelling's theories are perhaps better thought of as radically different explanations of a common phenomenon". Nonetheless, they highlight an opposition between

Schelling and Bacharach only on a superficial point, which in my opinion tends to hidden a much deeper issue. They oppose the fact that, according to Bacharach, “the existence of determinate solutions should *not* be treated as axiomatic” (*ibidem*) to the fact that, according to Schelling there is an “assumption that coordination games have determinate solutions” (*ibid.*, 25). But, fundamentally speaking, in my opinion this opposition concerns the way Schelling and Bacharach conceived players’ rationality.

In fact, in Schelling’s conception of rationality “players do not use the individualistic form of best-response reasoning represented in conventional game theory.” (Sugden, Zamarròn, 2006:614). Thus “the conception of rationality that we are attributing to Schelling is *not* the instrumental rationality on which conventional game theory is grounded. [...] In contrast, Schelling imposes no restrictions of validity on players’ reasoning” (*ibid.*, 620). My analysis shows that it is because social dimensions are necessary to cope with games and obtain sensible results that, instrumental and individual forms of rationality, as traditionally included and formalized in game theory, are no longer sufficient. By taking into account these social dimensions (that is, institutions, conventions, etc. already existent or built by the game), players think of themselves as members of a group. It is because of this identity feeling (*i.e.* the acknowledgement that they belong to a group) that they are able to understand in the same way the ‘clues’ given by the context of the interaction. Some institutions, that are preexistent or built in action, give them ‘clues’ to coordinate. They integrate into their reasoning social components which are recognized by themselves as members of this group. In fact, the way members of a group interact is specific to this group. This explains why individualism is breaking down. The other is no longer seen as an event which implies uncertainty and insecurity. Somehow, ‘the other’ gives information about his identity and about the way they, the players, will be able to find a solution. During their interaction, and because of their common identity, a solution can be found.

Furthermore, Gilbert (1989, 1990) to some extent justified Schelling’s intuition to overpass traditional game theory’s rationality – and further individualism – by asserting that it is impossible to justify the existence of a rationality which allows individuals to reach the focal point in a coordination game or in a cooperation game. And, in fact, it seems that Colman (1997: 13-14) uses this same argument. In reality, in a formal game theoretic apparatus there is still an infinite regression concerning players’ beliefs about other players’ beliefs, and no one has a rational basis to firmly imply that one of the outcomes is the solution. In addition, if social dimensions and non-mathematical dimensions of one of the outcomes must be introduced to solve the game, consequently, it is no longer possible for players’ rationality to be defined in the traditional way privileged by game theory. This again justifies why Schelling’s account of rationality bypasses traditional, individual and instrumental forms of rationality in games.

Schelling’s breakthrough on rationality can also be found in his focal point principle. By justifying the existence of the focal points through “ordinary folks” (Schelling, [1960] 1980: 58) or by the “the stability of institutions and traditions ...” (*ibid.*, 91), a kind of social form of rationality can be mentioned. Thus, conventions and institutions create focal points and render a solution unique, conspicuous and reliable (through coordinated expectations, and

reliability of these expectations). He refers too to “The force of many rules of etiquette and social restraint” (*ibidem.*) in order for players to coordinate. Schelling (*ibid.*, 92) explicitly asserted that social dimensions like people’s position in society, or more specifically in the precise coordination situation they are facing – in other words, to some extent people’s identity condition – participate in the convergence of expectations, and finally to the attainment of a solution (i.e., of the focal point). From my viewpoint, again it means that some aspects of the players’ identity, when the latter have to coordinate, intervene in their reasoning, and clearly this is not an individual but a social form of rationality. Furthermore, the introduction of an intersubjective capacity for players to coordinate in order, in some circumstances, to reach the focal point leads inevitably to weaken the role of individualism or even to bypass individualism. Players must put themselves in others’ shoes. To some extent they have to go beyond their own individuality to become, at least partially, another person.

Now concerning Bacharach, even if he asserted that frames are shared in a common culture²³ and that frames are culturally-based²⁴, he didn’t really explain and justify salience via these facts, although “VU-games may therefore be seen as a tool for giving precise expression to the emerging view [...] that the rational solutions of some games depend on the culture of the players” (Bacharach, 1993: 271). Finally, Bacharach’s approach however remained absorbed by the traditional way of reasoning in game theory. It remained confined to traditional individualism as it is formalized in the EUT (even if the concept of framing was added). In VFT, salience is explained in terms of risky choice. Players still maximize their expected utility and still maintain somewhat high computational capabilities to entail a high level of beliefs (i.e., beliefs about others’ beliefs about others’ beliefs, etc.)²⁵. This is obvious when Bacharach (in Bacharach, Bernasconi, 1997: 39) notes that “the essential depth assumption of VFT is that players think about salience strategically: i asks herself how likely it is that j has noticed what she has and so has the same options as her”. There is no intersubjectivity (contrary to Schelling) and there is no social dimension players’ rationality. Furthermore, when Bacharach argues (*ibid.*, 38) that saliency does not need to be shared in order to find a solution to a coordination game, we again understand that no social dimension, such as institutions, conventions, etc., can serve as a focal point (i.e., as a solution). Even if players belong to the same culture, Bacharach and Bernasconi (1997: 39) advise these real players that they can fall into a ‘*false consensus*’. In this case, primary salience is wrongly assumed to be shared. Players, in this context, cannot be influenced by a common background; they cannot see the same salient characteristics in the given situation and as a result, they fail to coordinate.

²³I must refer, for example, to these quotations: “An important stylized fact about frames is that within a culture framing propensities are largely shared. People tend to represent in the same way.” (Bacharach, 2001: 8) and “within a same culture the parameters of a conceptual scheme – its membership, its clustering, and the readiness to mind of the clusters in a given situation – are essentially shared” (Bacharach, 1993: 259)

²⁴“The membership, and the probability of “firing”, of clusters of concepts are (stylized) facts about players’ culture” (*ibidem*)

²⁵By the way, this infinite regression is unnecessary in Schelling’s account of focal points.

Consequently Schelling's accounts of game theory and of focal points are sustained by his particular conception of rationality. This explains why Bacharach's approach of game theory and of focal points appears to be so different from Schelling's one: Schelling bypasses the *individualism* in game theory and Bacharach does not.

6. Conclusion

In this contribution, the conjecture that Schelling's insight into the *refinement program* was a kind of *revisionary theory* has been confirmed, as well as the intuition that, by contrast, Bacharach's insight was just a simple extension. In summary, Bacharach's VFT belongs to the tradition (with: Sugden, 1995; Casajus, 1998; Janssen, 2001, 2006) that tried to justify the relevance of focal points in common interest games, in a rational way. This is indeed a strange statement because, in view of many aspects of his work, Bacharach was an innovative game theorist. He turned before his death to the conceptualization of collective rationality or of 'we thinking' in games. The most interesting fact is that he based his theory of 'we thinking' on the players' identity. Indeed, in his view, agents perceive themselves as members of a collective entity, a team, when they have a common interest to do something, when they face a common fate. Although Bacharach did not go as far as Schelling in a *revisionary theory*, he too belongs to this *refinement program* by investigating the epistemic foundations of game theory and by accounting for players' frames. However, Bacharach's VFT is finally closer to a form of '*respecification theory*'.

Now, some issues which fall outside of the subject matter of this analysis should be emphasized within Schelling's theory. They contribute to emphasize the fundamental interest of Schelling's account on game theory.

The first point I would like to mention is the similarity between Schelling's account of game theory and the contemporary *psychological game theory*²⁶. When Schelling argues that the way players play games provides information and that, to some extent, the strategy which is chosen by a player in the set of all the available strategies reveals information concerning his intentions and consequently influences the others' responses, this is precisely the heart of *psychological game theory*.

Another interesting aspect inferred by this comparison between Schelling account of game theory and *psychological game theory* is that, according to Colman (2004) and Hargreaves Heap and Varoufakis (1995, [2004]), the theories of *collective rationality* in game theory (Bacharach, 1999, 2001, 2006; Sugden, 1993, 2003, 2006), belong to the *psychological game theory*. Though the focal points are outside the scope of collective rationality in Bacharach there are perhaps more similarities between Schelling's vision of game theory and

²⁶For a general review of the psychological game theory, please refer to Hargreaves Heap and Varoufakis ([1995], 2004).

Bacharach's account on collective agency²⁷. Besides, as I have previously highlighted it, there is to some extent a kind of collective agency in a few aspects of Schelling's work. The question of the existence of groups and of what is a collective in game theory therefore provides a path for future investigation in Schelling's works.

Our last issue will concern the link between information and knowledge in game theory. Bacharach (1987:22) emphasized that information is currently assumed to be knowledge in games. It seems to me that in Schelling, knowledge is more than simple information. Players know each other, use artifacts, give sense to facts, to institutions, actions, etc. It is question-begging. Schelling suggested that people who know each other should be considered as a kind of collective entity. Once again we understand that, in his interpretation of game theory, Schelling went beyond the limits of methodological individualism and of usual economic rationality.

Bibliography

Bacharach M., (1976) *Economics and the Theory of Games*, The Macmillan Press LTD, London, UK

Bacharach M., (1987), "A Theory of Rational Decision", *Erkenntnis*, Vol. 17, No. 1, p 17-55

Bacharach M., (1993), "Variable Universe Game", in *Frontier of Game Theory*, Ed. by Binmore K., Kirman A., Tami P., The MIT Press, p 255-275

Bacharach M., (1997), "We Equilibria: A Variable Frame Theory of Cooperation", Institute of Economics and Statistics, University of Oxford

Bacharach M., (1999), "Interactive Team Reasoning: A Contribution to the Theory of Cooperation", *Research in Economics*, Vol. 53, p 117-147

Bacharach M., (2001a), "Superagency: Beyond Individualistic Game Theory", Department of Economics, University of Oxford, invited lecture presented at "TARK VIII: Eighth Conference on Theoretical Aspects of Rationality and Knowledge", Certosa di Pontignano, University of Siena,

Bacharach M., (2001b), "Framing and Cognition in Economics: the Bad News and the Good", Department of Economics, University of Oxford, paper presented at "ISER Workshop XIV: Cognitive Processes and Rationality in Economics", Certosa di Pontignano, University of Siena

²⁷I do not share the same conception of Bacharach theory of collective agency in game theory. If I acknowledge that Sugden's theory of team reasoning belong to psychological game theory, it is according to me not the case for Bacharach's theory of 'We thinking' (for this argument you must refer to Arena, Larrouy, 2012).

- Bacharach M., (2006), *Beyond Individual Choice ; Team and Frame in Game Theory*, Ed. by Gold N., Sugden R., Princeton University Press
- Bacharach M., Bernasconi M., (1997), “The Variable Frame Theory of Focal Points: An Experimental Study”, *Game and Economic Behavior*, Vol. 19, p 1-45
- Bacharach M., Hurley S. (1991) “Issues and Advances in the Foundations of Decisions Theory”, in *Foundations of Decision Theory*, Ed; by Bacharach M; and Hurley S., Blackwell Publisher, Oxford, UK
- Bacharach M., Stahl D. O. (2000), “Variable-Frame Level-n Theory”, *Games and Economic Behavior*, Vol. 32, p 220-246
- Bicchieri C., (1993), *Rationality and Coordination*, Cambridge Studies, in Probability, Induction, and Decision Theory, Ed. by Skyrms B., Cambridge University Press, New York, US
- Bradsley N., Mehta J., Starmer C., Sugden R., (2010) “Explaining Focal Points: Cognitive Hierarchy Theory *Versus* Team Reasoning”, *The Economic Journal*, Vol. 120, p 40-79
- Casajus A., (1998), “Focal Points in Framed Strategic Forms”, *Game and Economic Behavior*, p 263-291
- Colman A. M., (1997), “Salience and Focusing in Pure Coordination Games”, Working Paper, Pub. in *Journal of Economic Methodology*, Vol. 4, p 61-81
- Colman A. M., (2003) “Cooperation, Psychological Game Theory and Limitations of Rationality in Social Interaction”, *Behavioral and Brain Sciences*, Vol. 26, p 139-198
- Colman A. M., (2006), “Thomas C. Schelling’s Psychological Decision Theory: Introduction to a Special Issue”, *Journal of Economic Psychology*, Vol. 27, p603-608
- Gauthier D., (1975), “Coordination”, *Dialogue*, Vol. 14, No. 2, p 195-221
- Gilbert M., (1989), *On social Facts*, Routledge, London
- Gilbert M., (2000), *Sociality and Responsibility, New Essays in Plural Subject Theory*, Rowman and Littmeifeld Publishers, Lanham
- Gilbert M., (2003) “The Structure of the Social Atom: Joint Commitment as the Foundation of Human Social Behavior”
- Giocoli N., (2003), *Modeling Rational Agents: From the Interwar Economics to Early Modern Game Theory*, Edward Elgar, Massachusetts, USA
- Gold N., Sugden R., (2007), “Theories of Team Agency”, in *Rationality and Commitment*, Ed. by Peter and Schmidt, Oxford University Press, p 281-312
- Hargreaves Heap S. P., Varoufakis Y., ([1995] 2004), *Game Theory; A Critical Text*, Second Edition, Routledge, London and New York

- Heath C., Ho B., Berger J., (2006), "Focal Points in Coordinated Divergence", *Journal of Economic Psychology*, Vol. 27, p 635-647
- Hurley S., (1989), *Natural Reasons*, New York: Oxford University Press
- Janssen M. C. W., (2001), "Rationalizing Focal Points", *Theory and Decision*, Vol. 50, p 119-148
- Janssen M. C. W., (2006), "On the Strategic Use of Focal Points in Bargaining Situations", *Journal of Economic Psychology*, Vol. 27, p 622-634
- Lewis D., (2002 [1969]), *Convention*, Blackwell Publisher, Oxford
- Metha J., Starmer C., Sugden R., (1994), "The Nature of Salience: An Experimental Investigation of Pure Coordination Games", *American Economic Review*, Vol. 84, p 658-673
- Nash J., (1950), "Equilibrium points in n-person games", *Proceedings of the National Academy of Sciences*, Vol. 36, p48-49
- Schelling T. C., (1980 [1960]), *The strategy of conflict*, Harvard University Press, Cambridge, Massachusetts
- Sen A., (2007) « Why Exactly is Commitment Important for Rationality? », *Rationality and commitment*, Ed. by Peter F. and Schmid H. B., Oxford University Press, New York
- Sugden R., (1991), rational Choice: A Survey of Contributions from Economics and Philosophy", *The Economic Journal*, Vol. 101, p751-785
- Sugden R., (1993) "Thinking as a Team: Towards an Explanation of Non Selfish Behavior", *Social Philosophy and Policy*, Vol. 10, No. 1, p 69-89
- Sugden R., (1995), "A Theory of Focal Points", *The Economic Journal*, Vol. 105 (May), p 533-550
- Sugden R., (2000), "Team Preferences", Working paper, Pub. in *Economics and Philosophy*, Vol. 16, No. 2, p 175-204
- Sugden R., (2001), "The Evolutionary Turn in Game Theory", *Journal of Economic Methodology*, Vol. 8, No. 1, p 113-130
- Sugden R., (2005), "The Logic of Team Reasoning", in *Teamwork – Multi-Disciplinary Perspectives*, Ed. by Nathalie Gold, Palgrave MacMillan, p 181-199
- Sugden R., (2007), "Collective Intentions and Team Agency", *Journal of Philosophy*, Vol. 104, No. 3, p 109-137
- Sugden R., Zamarron I. E., (2006), "Finding the Key: The Riddle of Focal Points", *Journal of Economic Psychology*, Vol. 27, 609-621

von Neumann J., Morgenstern O., (1944), *The Theory of Games and Economic Behaviour*,
Princeton, NJ: Princeton University Press

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