WAS HARROD RIGHT?

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Abstract

of

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by

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Modern growth theory derives mostly from Robert Solow's "A Contribution to the
Theory of Economic Growth" (1956). Solow's own interpretation locates the origins of
his "Contribution" in his view that the growth model of Roy Harrod implied a tendency
toward progressive collapse of the economy. He formulates his view in terms of Harrod
invoking a fixed-coefficients production function. This paper, first, challenges Solow's
reading of Harrod, arguing that Harrod's object in providing a "dynamic" theory had little
to do with the problem of long-run growth as Solow understood it, but instead addressed
the medium run fluctuations. It was an attempt to isolate conditions under which the
economy might tend to run below potential. In making this argument, Harrod does not
appeal to a fixed-coefficients production function – or to any production function at all,
as that term is understood by Solow. The paper next traces the history of the dominance
of Solow's interpretation among growth economists. These tasks belong to the history of
economics. The paper's final task belongs to economic history. It offers an informal
reexamination of economic history through the lens of Harrod's dynamic model, asking
whether there is a *prima facie* case in favor of Harrod's model properly understood.

Was Harrod Right?[1]

One of the frustrations of historians of economics is that economists of today often have historical ideas that reveal a failure to have read the originals or to consider their meaning. It is not so much that economists do not care about the history of their discipline; rather they are willing to take its history as having been faithfully transmitted through the textbooks. They are apt to take positions on history that are strong nearly in proportion to their being wrong. I was brought up as an economist with such a strong view of Roy Harrod's "growth" model.

In their textbook, Robert Barro and Xavier Sala-i-Martin (1995, p. 10) write that Harrod "used production functions with little substitutability among the inputs to argue that the capitalist system is inherently unstable." Nearly every textbook that mentions Harrod in the context of growth – and many ignore him altogether – takes a similar line. Indeed, it is the story that I have myself told (Hoover 2003, p. 413). Harrod, it is said, offered a model of long-term economic growth that ignored substitution of factors of production and, therefore, possessed an unreasonable "knife-edge" property in which any step away from the warranted rate of growth led inexorably to collision with a full employment ceiling or to mass unemployment and depression. Robert Solow (1956), believing that that we should not ignore substitution in the long run, changed Harrod's production function from a fixed-factors to a flexible-factors form and showed that the price system would align the warranted rate of growth with the natural rate of growth, given by the growth of population and technology, and guarantee that deviations of the

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[1] This is a preliminary exploration of Harrod's "growth" model – one that is based heavily in two texts, Harrod's (1939) and Solow's (1956) essays, rather than in mastery of the scholarly literature on Harrod. I hope in time relate it more carefully to the wider literature.

warranted rate from the natural rate would be self-correcting. Far from being balanced on a knife edge, long-term growth was stable.

Reading Harrod's (1939) and Solow's (1956) essays side by side for a class that I taught on the history of modern macroeconomics convinced me that this potted history is a misreading, and that the misreading is one that has not grown much in the telling and the retelling, but one that, in fact, starts with Solow's essay itself. A reconsideration of Harrod shows that he was interested in very different questions than Solow and that only when one ignores that fact does it appear that he is obviously wrong. Indeed, Harrod offers a conjecture about the behavior of the economy that is not only hard to understand in Solow's framework but has also, to my knowledge, never really been given a careful empirical examination.

I. A Map of Misreading

Harrod's analysis provides the foil against which Solow displays the power of his own simple model of long-run economic growth. And by now, Solow's has become the canonical account of Harrod's model. Harrod's model pointed to pervasive instability in macroeconomic dynamics, which Solow characterized with a compelling metaphor: "the long run . . . economic system . . . balanced on a knife-edge of equilibrium growth" (1956, p. 65).[2] Solow casts his own model as the repudiation of Harrod's knife-edge. He locates the source of the knife-edge property in Harrod's assumption that production takes place under conditions of fixed proportions of factor inputs and claims to accept

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[2] When no confusion arises, references to Solow (1956) and Harrod (1939) are by page number only, omitting author and publication date.

"all the Harrod-Domar assumptions" except fixed proportions in developing his own

growth model, which does not display a knife-edge (p. 66).

We can see what Solow regards as Harrod's (and Domar's) assumptions simply

by looking at what he himself assumes for his own model (Solow 1956, pp. 66-68):

1) an economy with single-commodity (Y);

2) a constant savings rate (s), so that savings $S = sY$;

3) a constant-returns-to-scale production function with smooth substitution between capital and labor;

4) labor that grows at an exogenous rate (n);

5) "no scarce nonaugmentable resource like land";

6) flexible prices and wages;

7) constant full employment of factors of production;

8) and closely related to this last assumption, the identity of *ex ante* and *ex post* investment, which guarantees the identity of *ex ante* investment and savings, allowing the accumulation of capital to be described by the savings function alone.

The difference between his model and Harrod's rests entirely, Solow claims, on

assumption 3. Let us see how Solow exposits Harrod's model. Harrod maintains a *fixed-*

proportions production function:

$$(1) \qquad\qquad Y = F(K, L) = \min\left(\frac{K}{a}, \frac{L}{b}\right),$$

where Y = output, K = capital, L = labor, and a and b are production parameters. This

function does exhibit constant returns to scale, but it does not allow substitution between

capital and labor. The production function can be recast into what has been called

subsequently "intensive form":

$$(2) \qquad\qquad y = f(r) = \min\left(\frac{r}{a}, \frac{1}{b}\right),$$

where $y = Y/L$, $r = K/L$ and $f(r) = F(K/L, 1)$.

Solow had previously worked out the dynamics of growth in any system with a

constant-returns production function, yielding his "fundamental equation" (p. 69):

(3) $\dot{r} = sf(r) - nr$.

Without providing a precise derivation, it is easy to see what is going on. The time rate

of change of capital per worker ($\dot{r} = \dfrac{dr}{dt}$) is the difference between savings per worker

and the amount of additional capital needed to outfit a growing number of workers with

the current rate of capital per worker (*nr*). In terminology that was coined later, *capital*

deepening ($\dot{r}$) is the difference between additions to capital in the form of savings and

capital widening (*nr*).

Figure 1 (closely related to Solow's Figure IV (p. 74)) uses equations (2) and (3)

to present the dynamics of growth graphically. Panel A shows production, which rises as

r increases at a rate of $1/a$ up to the point that $r = a/b$, where it becomes horizontal at an

output $y = 1/b$. To the left of a/b, capital constrains output and there is some

unemployment of labor; to the right of a/b, labor is fully employed and additional capital

is redundant. The savings function *sf(r)* is a simple scaling of the production function. It

also has a kink at a/b where savings per worker $S/L = s/b$. The needs of capital widening

are shown as rays from the origin with a slope *n*. Consider the particular growth rate of

the labor force n_1. Its ray, labeled n_1r, intersects the savings function at $r = \dfrac{s}{n_1 b}$.

According to equation (3), the difference between the savings function and the

capital-widening ray determines the time rate of change of capital, which is shown as the

phase diagram in panel B. The difference reaches a maximum at $r = a/b$, falls to zero at,

4

and then become negative to the right of $r = \dfrac{s}{n_1 b}$. The arrows indicate that, for any value

of r less than $\dfrac{s}{n_1 b}$, r increases and, for any value greater, r decreases. While the phase

diagram is cast in terms of the time rate of change of r, the growth of output per worker

($\dot{y}/y$) is itself completely determined by the time path of r.

The growth rate of the labor force may be high enough (or, equivalently, the

savings rate may be low enough) that the nr ray never intersects the savings function.

The ray $n_2 r$ illustrates such a case, and panel C shows the corresponding phase diagram.

In such cases, r collapses toward zero, no matter what positive value it takes at the start.

Solow relates his model to Harrod's in the following way. First, for Harrod

(1939, p. 30), the natural rate of growth is given by the maximum rate permitted by the

rates of growth of population, technology, and labor-force participation. Thus, in a

model without technical progress and a constant participation rate, the natural rate of

growth $G_N = n$ unambiguously, and the slope of the nr ray is, in fact, the natural rate of

growth. Solow interprets Harrod's warranted rate of growth as the growth rate dictated

by the production function and the savings propensity. From equation (1), the time rate

of change of output $\dot{Y} = \dot{K}/a$. Since $\dot{K} = sY$, the warranted rate of growth

$G_W = \dot{Y}/Y = \dfrac{sY/a}{Y} = \dfrac{s}{a}$. And since the incremental capital-output ratio (C) is the inverse

slope of a ray from the origin to the point of production, $C = a$ for the production

function (1). Substituting yields Harrod's own "Fundamental Equation":

(4) $$G_W = \dfrac{s}{C}.$$

The slope of the savings function to the left of *a/b* in Figure 1, panel A, therefore, gives us the warranted rate of growth.

To understand Solow's analysis of the knife-edge, consider a case in which the capital widening ray n_3r coincides with the upward-sloping portion of the savings function as shown in Figure 1, panel A (the corresponding phase diagram is in panel D). Start with the economy at a full-employment, moving equilibrium in which it is growing at a warranted rate exactly equal to the natural rate ($G_W = G_N$). Any change of parameters that breaks the equality of the warranted and natural rates results in divergent movements of *r*. For example, an increase in the growth rate of labor to, say, n_2 (equivalent to a decrease in *s*) moves the economy from the phase diagram in panel D to that in panel C, which indicates the inexorable collapse of *r*, as the economy *relatively* disinvests.[3] On the other hand, a decrease in the growth rate of labor to, say, n_1 moves the economy to phase diagram B. The growth rate of output is unaffected, but redundant capital accumulates until the economy comes to rest again at $r = \dfrac{s}{n_1 b}$. In either of these extreme adjustments, the warranted and natural rates are permanently pulled apart. And any step away from their initial equality results in one of the extreme adjustments, which, according to Solow, define the knife-edge.

Solow attributes the knife-edge phenomenon to the fixed-proportions production function. To see why consider the analogous diagram to Figure 1 when capital and labor are smoothly substitutable. In Figure 2, panel A, neither the production function nor the savings function has a kink, and both are concave to the *r* axis (compare to Solow's

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[3] This is relative to the size of the labor force because capital is permanent and there is no depreciation in the model.

Figure I (p. 70)). The capital-widening ray n_1r is shown cutting the savings function

from below. The positive intersection at $r = r^*$ corresponds to a growth rate n_1. The

warranted rate of growth is given by the slope of a ray from the origin to the intersection

of $sf(r)$ and n_1r. Since this ray must coincide with n_1r, the warranted and natural rates of

growth are equal. The phase diagram in panel B shows that any small deviation of r from

r^* will reconverge on r^*; and, although the warranted and natural rates of growth may

temporarily diverge, they cannot be pulled permanently apart. Any small change in n or s

changes the location of r^*. Again, warranted and natural rates of growth would

temporarily diverge, but would reconverge on n over time. There is no excess capital at

r^*.

     While there is no knife-edge in Figure 2, Solow does not overclaim: "There *may*

not be – in fact in the case of the Cobb-Douglas function there never can be – any knife-

edge" (p. 73, emphasis added). A knife-edge could still occur if the capital-widening ray

rose faster at the origin than the savings function as shown with the ray n_2r in Figure 2

(compare to Solow's Figure III (p. 72)). The phase diagram in panel C shows that for

any initial r, progressive collapse towards zero capital and output is the only possible

outcome. Such a knife-edge cannot occur with the Cobb-Douglas production function:

since it has an infinite slope at the origin, any ray with a finite, positive slope must cut it

from below.[4]

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[4] The "Inada conditions" later provided a set of sufficient regularity conditions to guarantee the existence, uniqueness, and stability of a well-behaved steady-state equilibrium in which no knife-edge phenomena can occur (Inada 1963).

II. Harrod Recovered

In this section, I shall establish that Solow's interpretation is a misreading. It is, however, not clear whether his interpretation itself reflects a reading of Harrod at all. For while, Solow (p. 65) refers to the "Harrod-Domar model of economic growth" and, on occasion, separately to Harrod's model (e.g., pp. 66, 74), he cites neither Harrod's (1939) essay nor any other specific work of Harrod (or Domar). This may, of course, simply reflect the citation practices of the 1950s, which are clearly different from today's. It is nonetheless not clear whether Solow was drawing on an understanding grounded in a close reading of Harrod's work or on one that drew on views of Harrod that were "in the air" at the time. Because I believe that the misreading of Harrod does not turn on subtle points, it is more likely that Solow offers a formalization based on interpretations of Harrod that were at some distance from Harrod's text, and that it is likely that it was constructed backwards from Solow's own growth model in an effort to show which modifications captured commonly accepted views of Harrod's claims. At this point, this is simply conjecture.

Where does Solow go wrong?

In one sense, everywhere. Solow claims that his model differs from Harrod's *only* in not assuming a fixed-proportion production function. In fact, one could make a case that Harrod does not subscribe to *any* of the assumptions numbered 1–8 in Section I above. Some of the differences are probably benign, reflecting the fact Solow is engaged in the construction of a tightly specified, formal model, whereas Harrod is engaged in a much less formal analysis in which he can afford to be less committed to precise

assumptions than Solow must be. I will, therefore, not nitpick, but concentrate on what

seem like fundamental issues.

The *first* fundamental difference is that Harrod and Solow address different

conceptual problems. Solow's article is entitled "A Contribution to the Theory of

Economic Growth," and that title accurately conveys that Solow's own model is a model

of long-run economic growth. Solow self-consciously distinguishes between Keynesian

pathologies and a world of perpetual full employment:

> Everything [in my model] is the neoclassical side of the coin. Most especially it is
> full employment economics – in the dual aspect of equilibrium condition and
> frictionless, competitive, causal system. All the difficulties and rigidities which go
> into modern Keynesian income analysis have been shunted aside. [Solow p. 91]

Solow singles out r^* as the "equilibrium," where r^* is the point at which, after all

adjustments are done, the economy achieves a steady rate of growth equal to the natural

rate – a long-run equilibrium – even though at every point along the adjustment path

supplies equal demands and saving and production plans are all satisfied, so that there is

never any disequilibrium in a wider sense.

Harrod's article, in contrast, is entitled "An Essay in Dynamic Theory." The

implicit contrast is not between short-run and long-run or between full employment

(neoclassical) and less than full employment (Keynesian) but between dynamic and

static:

> Static theory consists of a classification of terms with a view to systematic
> thinking, together with the extraction of such knowledge about the adjustments due
> to a change of circumstances as is yielded by the "laws of supply and demand.". . .
> [Dynamic] "theory" would not profess to determine the course of events in detail,
> but should provide a framework of concepts relevant to the study of change
> analogous to that provided by static theory for the study of rest. [Harrod p. 14]

Harrod defines "dynamic" broadly "as referring to propositions in which a rate of growth appears as an unknown variable" (p 17). Thus, dynamics includes more than models of economic fluctuations, such as formal multiplier-accelerator models in which dated variables and explicit lags play a crucial role. He regards such models as perhaps explaining oscillations about trends, but he also suggests that oscillations in the trend itself are a crucial part of dynamics (p. 15). In contrast to Solow's equilibrium at r^*, in which growth had settled in to a steady rate, Harrod holds that

> [t]he line of output traced by the warranted rate of growth is a moving equilibrium, in the sense that it represents the one level of output at which producers will feel in the upshot that they have done the right thing, and which will induce them to continue in the same line of advance. [Harrod, p. 22]

The equilibrium is moving, not only in that the economy on a warranted path is growing, but also in that the parameters that govern the warranted rate itself may change frequently without any sense of convergence to a steady-state "equilibrium" of Solow's type. Indeed, Harrod's object is not to analyze a particular path *ceteris paribus* for the economy, not to compare the values of variables at different periods, but to elucidate the forces that may systematically drive the economy away from its equilibrium, warranted path *at any particular point of time* (pp. 17, 24-25). Growth is a central concern, since growth and change are essentially synonyms, but *long-run* economic growth of the type that animates Solow simply does not define the agenda.

The *second* fundamental difference between Solow's interpretation and Harrod's essay is that Harrod makes no explicit assumptions about a production function, and his implicit assumptions do not warrant a fixed-proportions, constant-returns-to-scale production function, such as equations (1) or (2). In the methodological preamble to his essay, Solow writes, "The art of successful theorizing is to make the inevitable

10

simplifying assumptions in such a way that the results are not very sensitive" (p. 65). His

assumption of an economy with a single good is, perhaps, one of these inevitable

simplifying assumptions, but it is not one that Harrod shared:

> [aggregate output is] compounded of all individual outputs. I neglect questions of
> weighting. Even in a condition of growth, which generally speaking is steady, it is
> not to be supposed that all the component individuals are expanding at the same
> rate. [Harrod, p. 16]

Harrod saw aggregate output as a summary statistic and not as a single

commodity. This might appear to be a small difference. Solow did not literally believe

that the economy produced a single commodity; rather he made a strong, simplifying

assumption. The importance of the difference becomes clear in Solow's derivation of the

knife-edge from the sharply defined parameters of the production function. The capital-

output ratio (C) is a fixed parameter (a in equations (1) and (2)) for Solow. It is not fixed

for Harrod:

> The value of C depends on the state of technology and the nature of the goods
> constituting the increment of output. It may be expected to vary as income grows
> and in different phases of the trade cycle; it may be somewhat dependent on the
> rate of interest. [p. 17]

> C may also be expected to vary with the size of income, e.g., owing to the
> occurrence of surplus capital capacity from time to time . . . [p. 25]

While the capital-output ratio is not constant for Harrod, it is importantly

independent of the actual rate of growth. Its independence is related to the distinction

between the actions of the individuals and their aggregate consequences, a well known

Keynesian trope that frequently appears in discussions of fallacies of composition

(Harrod pp. 22-25). While the aggregate balance is governed by the warranted rate of

growth (and, therefore, by C and s), the actual rate of growth is governed by the reactions

of many individual producers to particular conditions of over- or under-production.

Individually rational responses to these particular conditions drive aggregate actual

growth away from the warranted path. *Harrod's instability* is, then, *pace* Solow, not an

unstable relationship between the warranted and natural rates of growth but an unstable

relationship between the warranted and *actual* rates of growth.

   Where Solow's own model involves an endogenous adjustment of the capital-

output ratio to that required by the natural rate of growth, endogenous adjustment is not

critical for Harrod. It is not, however, that he denies the possibility. *C* is not a constant;

it may respond over time to excess capacity or other factors. Still, what matters to Harrod

is its value at a point of time. He is not primarily interested in the specific paths of output

or in the long-run consequences of some change in parameters or policy action. Rather

he wants to point out the difficulty in staying on a warranted path at any moment. Where

Solow emphasizes the ultimate consequences of the knife-edge, Harrod emphasizes only

how hard it is to stand on such a narrow support.

   The second fundamental difference between Solow's interpretation and Harrod's

essay is closely related to a *third* – and probably most fundamental – difference: for

Solow *ex ante* savings and investment are always equal to *ex post* savings and

investment. "A remarkable characteristic of the Harrod-Domar model," Solow writes "is

that it consistently studies long-run problems with the usual short-run tools" (p. 66). This

is not a characteristic of Harrod's analysis, remarkable or otherwise, because, as I already

showed, Harrod is not concerned with long-run problems, but with the instability of the

actual growth rate relative to the moving equilibrium (warranted) growth rate – a short-

run problem for which he believes short-run tools are appropriate. Solow contrasts the

warranted and the natural growth rates; Harrod mainly contrasts the warranted and the

actual rates. It is seventeen pages into a thirty page essay before Harrod so much as

mentions the natural rate of growth, and, by that point, the core analysis is complete.

Harrod defines the incremental capital-output ratio (*C*) as

> that addition to capital goods in any period, which producers regard as ideally
> suited to the output which they are undertaking in that period. . . [T]he term *ex ante*
> . . . will be used in this sense. [p. 19]

Harrod proceeds in the standard Keynesian manner: when *ex post* investment falls short

of *ex ante* investment, output is stimulated as producers react to unexpected and

undesired reductions of inventories ("stocks" for Harrod), and conversely when *ex post*

investment exceeds *ex ante* investment. Harrod's principal mechanism is "a marriage of

the 'acceleration principle' and the 'multiplier' theory" (p. 14). It is not, as Solow

suggests, the wrong tool for the job; it is a different tool for a different job.

While Solow is aware that he has banished Keynesian problems from his own

model by assuming constant full employment and the equivalence (not just the *ex post*

equality) of savings and investment, he does not acknowledge that it is this assumption

rather than the substitutability assumption that ultimately separates his model from

Harrod's theory (Solow, p. 91).

The key role of the assumption that *ex ante* and *ex post* investment are constantly

equal can be clarified by reflecting on so-called optimal growth models, now a standard

part of the first-year graduate curriculum in economics (see Blanchard and Fisher 1989,

ch. 2, for one of many textbook expositions). Optimal growth models are essentially

Solow's neoclassical growth model in which the savings rate is no longer given

parametrically, but is the endogenous outcome of an intertemporal utility-maximization

problem. The phase diagrams for such models typically involve not just capital per

worker (*r*) but also consumption per worker. And typically, they are everywhere unstable

except for a unique saddle path. The perfect foresight, or rational-expectations, solution

simply assumes that a rational agent would ignore all other paths and jump to the saddle

path. The saddle path is precisely the set of points for which *ex ante*, planned investment

and consumption decisions align with *ex post*, realized investment and consumption.

Growth along the saddle path is growth at the warranted rate, even though that rate (as it

does on the phase diagram in Figure 2, panel B) changes constantly until it reaches the

steady state. But what happens if we do not assume perfect foresight or rational

expectations? Then, the economy is, except by sheer luck, in the unstable part of the

phase space. The actual growth rate differs from the warranted rate – an instability of

Harrod's, not Solow's, form, despite the fact that the production function is one with

smooth substitutability.

What if we maintain perfect foresight or rational expectations but replace the

production function with a fixed-proportions function such as equation (2)? The model

still determines a phase space unstable everywhere except a saddle path, and the solution

is to jump directly to the saddle path. The key assumption is not the substitutability of

factors. The key assumption is the equivalence of *ex ante* and *ex post* investment.[5]

The *fourth* fundamental difference between Solow's interpretation and Harrod's

essay is that Harrod is that Solow's knife-edge and Harrod's instability along the

warranted growth path are almost completely unrelated ideas. As Solow describes it:

> Were the magnitudes of the key parameters – the savings ratio, the capital-output
> ratio, the rate of increase of the labor force – to slip ever so slightly from dead

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[5] This point has not completely escaped notice: According to Burmeister and Dobell (1970, p. 41),
"studying the Harrod position as if it were based essentially on a technological hypothesis about the
production function, namely that it shows fixed proportions, misses the essential feature of Harrod's
analysis."

center, the consequence would be either growing unemployment or prolonged inflation. [p. 65]

As we already seen, a knife-edge phenomenon is not unique to the fixed-proportions production function. Compare the phase diagrams (panels B and C) in Figures 1 and 2. They are topologically identical. In each case, if the *nr* ray rises faster at the origin than the production function (panel C), then there is a progressive collapse until growth and production disappear. In each case, if the *nr* ray cuts the savings function from below, there is progressive expansion until technology limits output to factor capacity and growth to the natural rate. There is one salient difference: in Figure 2 with substitutable factors, full employment is maintained along the adjustment path; while in Figure 1, unemployment rises in panel C and capital is unemployed at the steady state in panel B. Of course, in Figure 2, panel C, unemployment is effectively 100 percent in the long-run state (at the origin), even if it is zero on the transition path. And if Harrod is to be believed and *C* adjusts to surplus capacity (p. 25), the excess capital in Figure 1, panel B, will not persist.[6]

The considerations so far demonstrate that some of Solow's knife-edge properties do not depend principally on the assumption of a fixed-proportions production function. In particular, whether the economy away from the natural rate of growth grows to an upper or lower bound appears to be independent of substitutability. Rather it depends on regularity conditions in the production function – at the lower end, principally whether the slope of the production function is vertical at the origin. The existence of surplus

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[6] It is hard to see why the steady state in Figure 1, panel B, should be characterized as contributing to "prolonged inflation," as Solow suggests, especially in a model in which there is no money and no general price level. There is not even excess demand at the steady-state. There is excess accumulation of capital, and that, rather than, inflationary pressure seems to be the proper analogy to the increasing unemployment in panel C.

factors of production is a difference that can be more clearly attributed to nonsubstitutability; but, as we have already seen, Harrod does not appear to subscribe to a fixed-proportions production function.

More fundamentally, Solow's knife-edge is not the same as Harrod's instability of the warranted rate of growth. Solow's analysis of the knife-edge concerns the failure of the warranted rate to adjust to the natural rate, where Harrod's instability concerns the divergence of the actual rate of growth from the warranted rate. The radical difference between them is clear from the fact that they work in opposite directions.

As we showed in Section I, if an economy with a fixed-proportions production function starts with equality between the warranted and natural rate and if that equality is upset, for example, by a fall in the savings rate (s), which lowers the warranted rate (s/C), then the economy as described in the phase diagram in Figure 1, panel C, crashes towards zero output. This is one side of Solow's knife-edge.

Consider, Harrod's analysis of the same fall in the savings rate. The fall in the savings rate reduces the warranted rate of growth below the actual rate of growth. "Savers will find that they have saved more than they would have done had they foreseen their level of income . . . Consequently they will be stimulated to expand purchases, and orders for goods will consequently be increased" (Harrod, p. 21), which widens the divergence between the actual rate of growth G and the warranted rate G_W. Whereas in the case of Solow's knife-edge, a fall in the savings rate directs the economy toward lower levels of output, in the case of Harrod's instability, it directs the economy toward higher levels of output. Solow's knife-edge and Harrod's instability concern the relationship of different growth rates (G_W and G_N for Solow; G and G_W for Harrod) and

16

for any change in parameters (*s*, *n*, *C*), they work in opposite directions. They address

nearly orthogonal issues.

Harrod's instability can arise in Solow's own model provided that *ex ante* and *ex post* investment are allowed to diverge. Consider the phase diagram in Figure 3, which

corresponds to panel B in Figure 2. In the usual analysis of Solow-type neoclassical

growth models, the economy is restricted to paths along the capital-adjustment curve.

Consider what happens when the economy is at the steady state *r** and the savings rate

falls. The curve shifts down, and normally the analysis proceeds by finding a point on

the new curve directly below *r**. This point is now out of steady state, but the arrows

along the curve indicate that capital would be relatively depleted until a new steady state

where $\dot{r} = 0$ was reestablished. But what justifies the jump from the initial point to the

new curve? Only the insistence that there be no divergence between *ex ante* and *ex post*

– rationalized by an assumption such as perfect foresight or rational expectations. If, as

Harrod presumes, we can make no such assumption, then the economy is left at a point in

the phase space ($r*$, $\dot{r} = 0$) that is above the new capital-adjustment curve, and all points

above this curve are driven to the right (and all below to the left) by the interaction of the

"marriage of the 'acceleration principle' and the 'multiplier'." The capital-adjustment

curve is thus seen to be a saddle-path, similar to what one typically finds with

neoclassical optimal growth models.

If we interpret, the capital-adjustment curve in Figure 3 as we did in Section I as

governing the transient movements in the warranted rate of growth as it adjusts toward

the natural rate, then Harrod can be said to have directly described the situation analyzed

using Figure 3 in terms that accurately reflect the way it is drawn. A fall in the savings

rate is a Keynesian stimulus. Harrod writes:

> Suppose that one of these stimulants begins to operate when the actual rate is equal
> to the warranted rate. By depressing the warranted rate, it drags that down below
> the actual rate, and so automatically brings the actual rate into the field of
> centrifugal forces, driving it away from the warranted rate-that is, in this case,
> upwards. Thus the stimulant causes the system to expand. [p. 31]

III. Harrod's Conjecture in Economic History

So far, I have argued that Solow's criticism of Harrod misfires because the key

difference between them is found not in assumptions about substitutability in the

production function but in the assumption that *ex post* and *ex ante* quantities are always

equal. Harrod's instability is, therefore, the instability of the actual growth rate relative

to the warranted rate, whereas Solow's knife-edge is the instability of the warranted rate

relative to the natural rate. Yet, Harrod does consider the relationship of the warranted

and natural rates; does that not lend some support to Solow's analysis? I think not.

Harrod considers the natural rate only late in the paper, after his main analytical

conclusions have been established. Solow locates the knife-edge phenomenon in any

case in which the warranted and natural rates fail to converge, so that the possibility that

there is a persistent gap between warranted and natural rates, which Harrod seems to take

as given, is exactly what Solow rejects. Yet, as we have already seen, there need be no

persistent gap for Harrod's instability to arise. The message of Figure 3 is that, even if

over time warranted and natural rates converge as in Solow's model, any change in

parameters that temporarily drives them apart sets up Harrod's instability – with or

without a fixed-proportions production function.

Still, it is worth considering Harrod's analysis of the relationship between the warranted and natural rates, since it has been neglected and it ends in a testable conjecture.

Solow is, of course, correct that Harrod does not see the convergence of the warranted and natural rates as given: "There is no inherent tendency for [the warranted and natural] rates to coincide" [Harrod, p. 30]. Their coincidence, Solow says, would be just "an odd piece of luck" on Harrod's view (p. 77). In one sense, that is right, but probably not for Solow's reasons. Since Solow sees the natural rate as evolving slowly and the warranted rate in his account of Harrod's model as governed by fixed parameters, only if those parameters happened to be just right could the two rates coincide. Harrod himself rejects the fixed parameter view: "Indeed, there is no unique warranted rate; the value of the warranted rate depends upon the phase of the trade cycle and the level of activity" (p. 30). Again, it is clear that Harrod's concern is not with long-run economic growth.

Unlike recent macroeconomic analyses that relate full employment or potential output to something like Milton Friedman's natural rate of unemployment and, so, allow for output to run above, as well as below, full employment or potential, Harrod's conception of full employment is the typically Keynesian notion of a ceiling. An economy is fortunate if it operates close to the ceiling, but it can never operate above it. Harrod introduces a fourth rate of growth to his famous three: actual, warranted, and natural. The *proper warranted rate of growth* is "that warranted rate which would obtain in conditions of full employment" (p. 30). While the economy cannot grow faster than the natural rate allows – that is, it cannot operate above full employment, even though it

may grow faster than potential output when it starts below full employment – Harrod

conjectures that the relationship between the proper warranted rate and the natural rate

determines the *likelihood* of the economy operating below full employment for any

length of time. He writes:

> The system cannot advance more quickly than the natural rate allows. If the proper
> warranted rate is above this, there will be a chronic tendency to depression; the
> depressions drag down the warranted rate below its proper level, and so keep its
> average value over a term of years down to the natural rate. But this reduction of
> the warranted rate is only achieved by having chronic unemployment.
>
> The warranted rate is dragged down by depression; it may be twisted upwards by
> an inflation of prices and profit. If the proper rate is below the natural rate, the
> average value of the warranted rate may be sustained above its proper level over a
> term of years by a succession of profit booms. [p. 30]

Harrod's thinking can be understood with the help of Figure 4. Figure 4 shows

the time path of output (Y) – logged values are used so that growth rates correspond to the

slopes of the curves. In panel A, up to time t_0, the proper warranted rate and the natural

rate coincide. At t_0, the proper warranted rate increases (s increases or C falls) as shown

by the upper, lighter path. Since the natural growth path is the full employment path,

actual output cannot move into the area between the natural and warranted growth paths.

Harrod's instability can manifest itself only in the downward direction as shown by the

arrows.

In contrast, panel B shows a case identical up to t_0, in which the proper warranted

rate falls at t_0 (s falls or C rise). Harrod's instability can still manifest itself downward,

but the region above the proper warranted growth path and below the natural growth path

is now feasible, so that it may also manifest itself upwards. Instability can, in this case,

drive the economy toward full employment and keep it there. Recessions are possible in

both cases; but, when the proper warranted rate exceeds the natural rate, they are

inevitable; and, when the natural rate exceeds the proper warranted rate, full employment may prove a common outcome. Harrod's conjecture, then, is that the economy will be more stable in the sense of spending more time near full employment when the natural rate exceeds the proper warranted rate than vice versa. As far as I know, there has yet to be an explicit test of the conjecture.

I now leave the history of economics to make a brief foray into macroeconomics – or, at least, into economic history. Harrod's conjecture says that an economy in which the proper warranted rate of growth (G_{PW}) stands below the natural rate of growth will display "a chronic tendency to depression" and an economy in which the proper warranted rate stands below the warranted rate may be frequently driven towards full employment. There are various ways of testing this conjecture. I will examine a particularly simple one.

The proper warranted rate of growth is that warranted rate which would obtain at full employment. Harrod's notion of full employment is, as we have noted already, a ceiling. In order to construct relevant growth rates, I first construct a potential output series for the United States 1929-2005. (The details of the data construction are found in the appendix.) The time series is generated from a Cobb-Douglas production function in which the inputs are the available labor force and the capital stock and the level of total factor productivity is estimated to grow smoothly along the upper bounds of actual total-factor productivity. From the time series for potential output and the capital stock, estimates of C and the net savings rate (s) can be constructed, finally yielding an estimate of the proper warranted rate for each period: $G_{PW,t} = \dfrac{s_t}{C_t}$. The natural rate ($G_N$) is just the

rate of growth of potential output at each period. The output gap is simply the percentage by which actual output fall short of potential output each period.

One implication of Harrod's hypothesis is that the output gap should be inversely related to the difference between the natural and the warranted rates ($G_N - G_{PW}$). The downward-sloping regression line through the scatterplot presented in Figure 5 shows that this is indeed the case: when the U.S. economy has had a proper warranted rate of growth that was low relative to the natural rate, it has operated nearer to full potential. A qualitatively similar result holds even if we consider only post-World War II data.

While this is a primitive test of Harrod's conjecture, it does, at the least, suggest that Harrod's now neglected analysis may have genuine empirical content and may be worthy of further, more careful investigation.

IV. Harrod's Fate

Solow's neoclassical growth model is a vastly important contribution to understanding the process of long-run economic growth. Solow originally cast his model as the solution to the problem of instability in Harrod's dynamic analysis. The economics profession generally accepted Solow's model, as well as his reading of Harrod, with the result that Harrod's analysis has been consigned to the dustbin of history. Barro and Sala-i-Martin (1999, p. 10), for example, write of Harrod's supposed fixed-proportions growth model: "Although these constructions triggered a good deal of research at the time, very little of their analysis plays a role in today's thinking." This is an understatement. Figure 6 shows the relative number of articles in the JSTOR archive that cite "Harrod" and "growth" relative to "Solow" and "growth," starting in 1939. It is easy to spot the high

tide of growth economics around 1970. Citations to both Harrod and Solow fall

substantially after that. What is less striking, but true nonetheless, is that even as

citations to both fall, citations to Solow *relative* to Harrod rise substantially. After 1985,

growth-related citations to Solow enjoy a substantial revival, probably related to the rise

of endogenous growth models and to the incorporation of the Solow growth model into

real-business-cycle models (see Hartley, Hoover, and Salyer 1998, ch. 1). By the end of

the period, growth-related citations to Solow outnumber those to Harrod ten to one.

One of the ironies of modern macroeconomics is that real-business-cycle models

use Solow's long-run growth model, in which the economy never deviates from the

warranted rate of growth, to explain short-run fluctuations. For Solow, this is precisely

the opposite of what he sees as Harrod's error: using long-run tools for a short-run

problem. Solow is, however, a short-run Keynesian, who agrees that savings and

investment decision need not be coordinated *ex ante.* To not address this issue in his

growth model was an analytical choice, not a claim that such problems did not arise in

the world:

> All the difficulties and rigidities which go into modern Keynesian income analysis
> have been shunted aside. It is not my contention that these problems don't exist, nor
> that they are of no significance in the long run. My purpose was to examine what
> might be called the tightrope view of economic growth and to see where more
> flexible assumptions about production would lead a simple model. Under-
> employment and excess capacity or their opposites can still be attributed to any of
> the old causes of deficient or excess aggregate demand, but less readily to any
> deviation from a narrow "balance." [Solow, p. 91]

Solow served advanced notice that he was unsympathetic to what, after the 1970s,

amounted to the collapsing of short-run into long-run analysis through appeals to rational

expectations: "No credible theory of investment can be built on the assumption of perfect

foresight and arbitrage over time" (p. 93).

The contention of this paper has been that, however valuable Solow's exploration of flexible assumptions has proved for the theory of long-run economic growth, Harrod was not the right target. His analysis was not of long-run economic growth, but of short-run or medium-run economic dynamics. He made no explicit assumptions about the flexibility, or lack thereof, of the production function, and gave substantial reasons to reject the idea that he thought that a fixed-proportions production function with constant parameters governed the economy. His instability was not Solow's knife-edge divergence of the warranted from the natural rate of growth, but a Keynesian divergence of the actual from the warranted rate of growth.

Was Harrod right? Such a question remains hard to answer. He did not make the analytical mistakes that Solow and subsequent economists have typically attributed to him. And there is some evidence – highly preliminary – in favor of his conjecture that an economy in which the proper warranted rate is high relative to the natural rate is likely to be recession-prone. Ultimately, these are not questions for the historian of economics, but for the empirical macroeconomist.

Appendix: Data Construction

BASIC DATA SOURCES.

Bureau of Economic Analysis website (http://www.bea.gov), interactive tables.
   Downloaded on 20 May 2008, last revised 30 April 2008. All data are annual, 1929-2005.

   Gross domestic product (*NY*): Source: line 1 of Table 1.1.5. Gross Domestic Product.
      Units: billions of dollars.
   Real gross domestic product (*Y*): Source: line 1 of Table 1.1.6. Real Gross Domestic
      Product, Chained Dollars. Units: billions of constant 2000 dollars.
   Compensation of Employees: Source: line 2 of Table 2.1. Personal Income and Its
      Disposition. Units: billions of dollars.
   Proprietor's Income: line 9 of Table 2.1. Personal Income and Its Disposition. Units:
      billions of dollars.
   Capital Stock (*K*): Source: Private and government fixed assets from line 14 of Table 1.2.
      Chain-Type Quantity Indexes for Net Stock of Fixed Assets and Consumer Durable
      Goods. Units: Index number, 2000 = 100, converted to constant 2000 dollars
      based on the value for 2000, $26,902.2 billion from line 14 of Table 1.1. Current-
      Cost Net Stock of Fixed Assets and Consumer Durable Goods.

Bureau of Labor Statistics website (http://www.bls.gov), interactive tables. Downloaded
   on 20 May 2008. All data are converted to annual from monthly data by averaging 1948-
   2005.
   Labor Force (*LF*, 1948-2005): Source: The Employment Situation, Civilian Labor
      Force, Persons 16 years of age and older, Series CLF16OV. Units: thousands.

Richard Sutch *et al.* (2006) *The Historical Statistics of the United States*, Millennial
   Edition. Cambridge: Cambridge University Press.
   Labor Force(*LF*, 1929-1947): Source: Table Ba470-477. Labor force, employment,
      and unemployment: 1890-1990 [Weir], Series BA470, Civilian labor force, total.
      Units: thousands. Note: 456 (thousand) added to this earlier series to butt splice it
      to the later BLS series.

CONSTRUCTION OF THE OUTPUT GAP.

Labor share in output (α): $\alpha_t = \dfrac{compensation\ of\ employees}{NY_t - proprietors'income}$.

   Subtracting *proprietors' income* amounts to assuming that *proprietors' income*
   divides into labor and capital income in the exact same way as the remainder of GDP.
   α = the mean value of $\alpha_t = 0.68$.

Total-factor productivity (*A*): $A_t = \dfrac{Y_t}{LF_t^{\alpha} K_t^{1-\alpha}}$.

   The use of the labor force (*LF*) rather than employment has the effect of incorporating
   the "inefficiency" of unemployment into *A*. Since we are ultimately interested in the
   efficient envelope of the A_t, there is not misleading.

Full employment total-factor productivity (A^{full}): A quadratic trend is fitted to A by ordinary least squares: $\hat{A}_t = (1.785 \times 10^{-5})\,time^2 - 0.063time + 56.063.$ $A_t^{full} = \hat{A}_t + 0.1.$

The additional constant has the effect of shifting the whole path of $\hat{A}_t$ so that it is forms the outer envelope of the A_t.

Potential GDP (Y_t^{pot}): $Y_t^{pot} = A_t^{full} LF_t^{\alpha} K_t^{1-\alpha}.$

GDP Gap (Gap_t): $Gap_t = \dfrac{Y_t^{pot} - Y_t}{Y_t^{pot}}.$

CONSTRUCTION OF THE NATURAL RATE OF GROWTH.

Natural Rate of Growth (G_N): $G_N = \dfrac{Y_t^{pot}}{Y_{t-1}^{pot}} - 1.$

CONSTRUCTION OF THE PROPER WARRANTED RATE OF GROWTH.

Trend capital ($\hat{K}$): An exponential trend is fitted to K by ordinary least square: $\hat{K} = \exp(-53.840 + 0.029time).$

Capital-output Ratio (C): $C_t = \dfrac{\hat{K}_t}{Y_t^{pot}}.$

Net Savings Rate (s): $s_t = \dfrac{\hat{K}_t - \hat{K}_{t-1}}{Y_t^{pot}}.$

Proper Warranted Rate of Growth (G_{PW}): $G_{PW,t} = \dfrac{s_t}{C_t}.$

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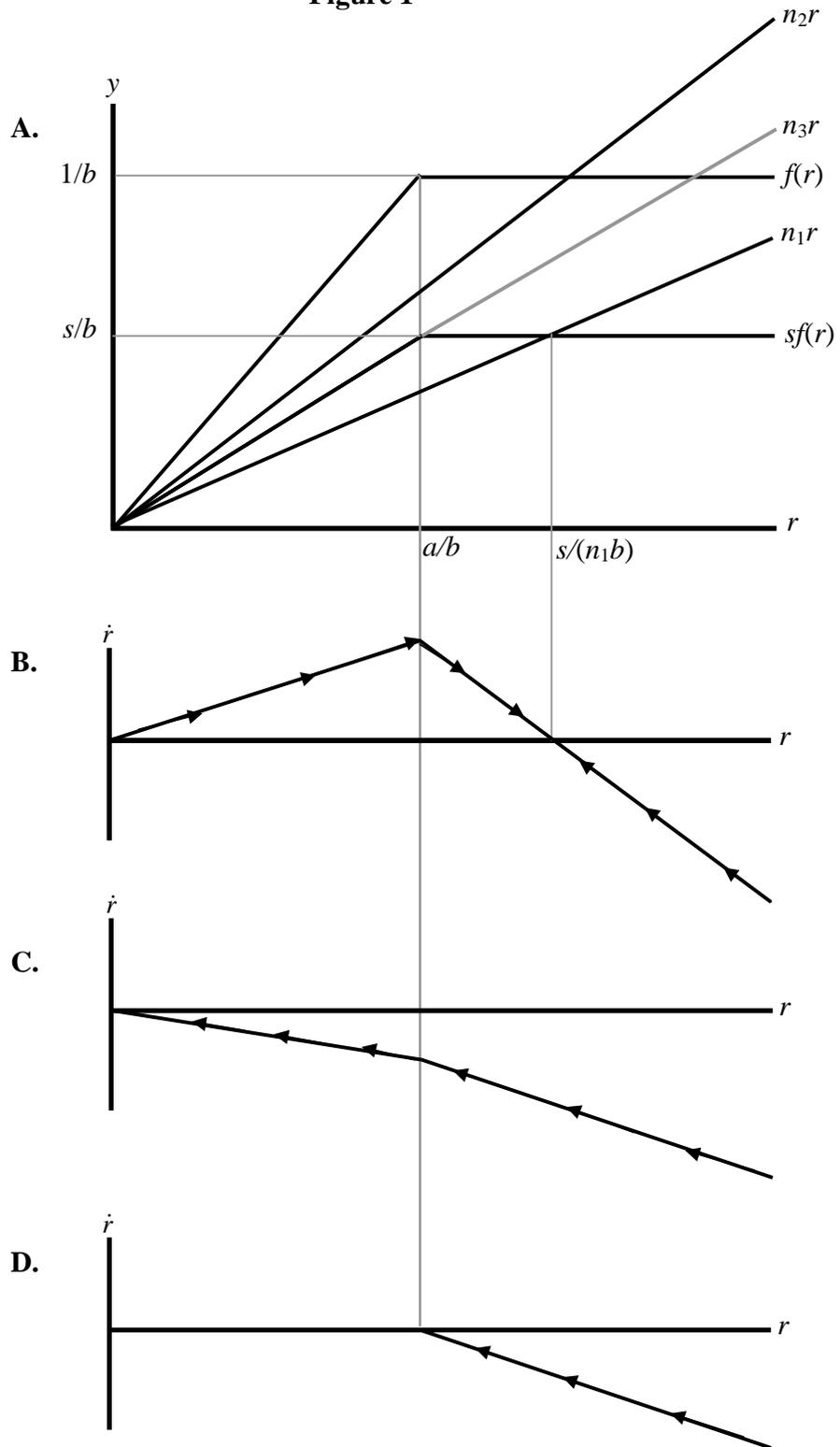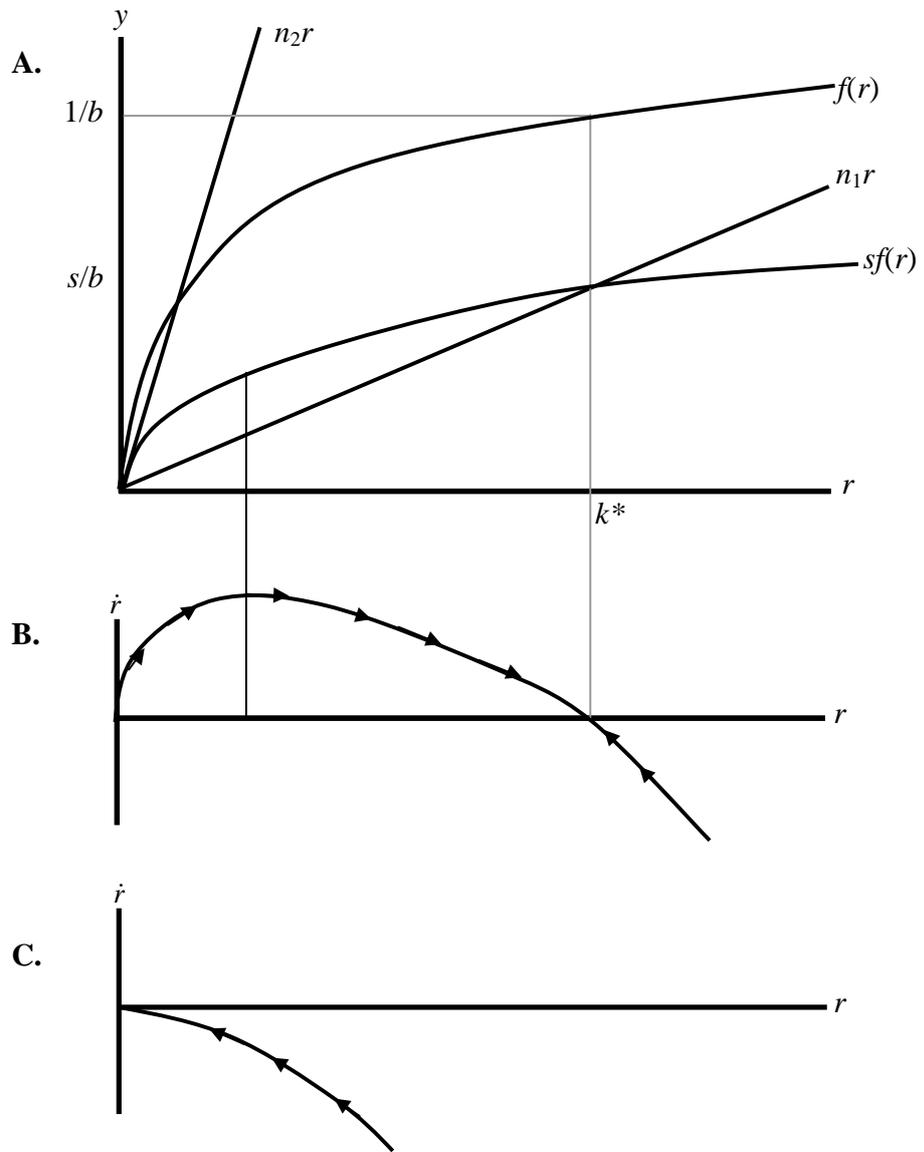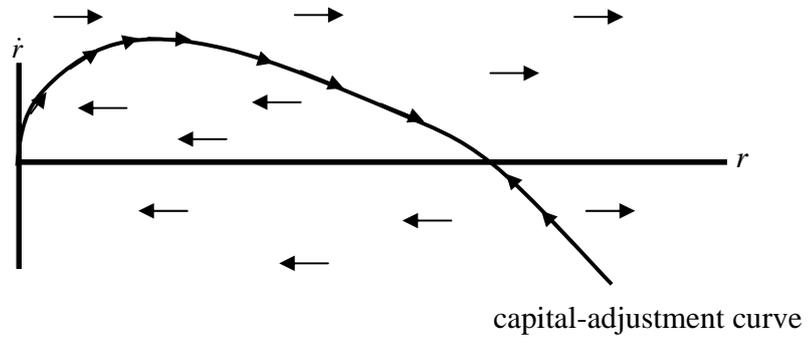
Figure 1

Figure 2

Figure 3.

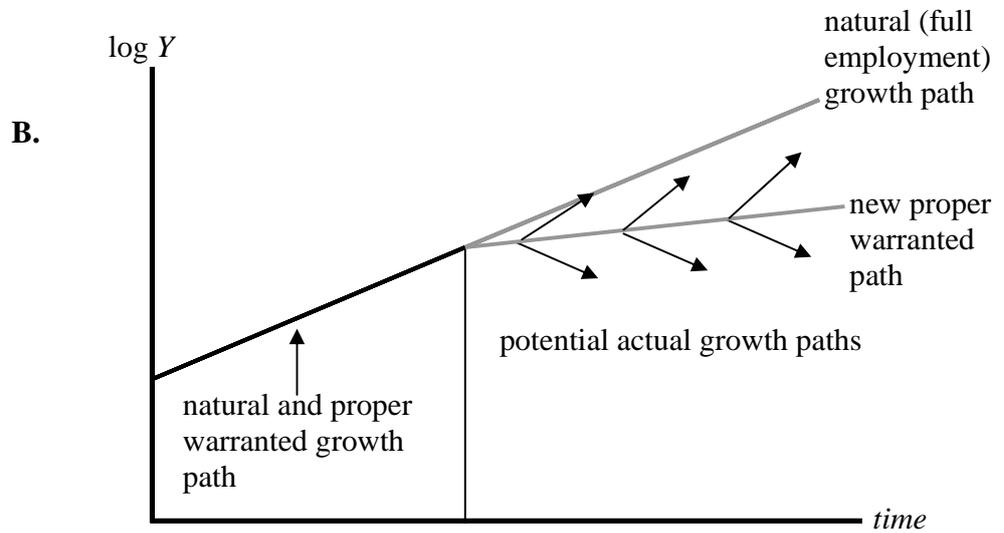capital-adjustment curve

Figure 4

A.

log *Y*

new proper warranted path

natural (full
employment)
growth path

potential actual growth paths

natural and proper
warranted growth
path

time

B.

log *Y*

natural (full
employment)
growth path

new proper
warranted
path

potential actual growth paths

natural and proper
warranted growth
path

time
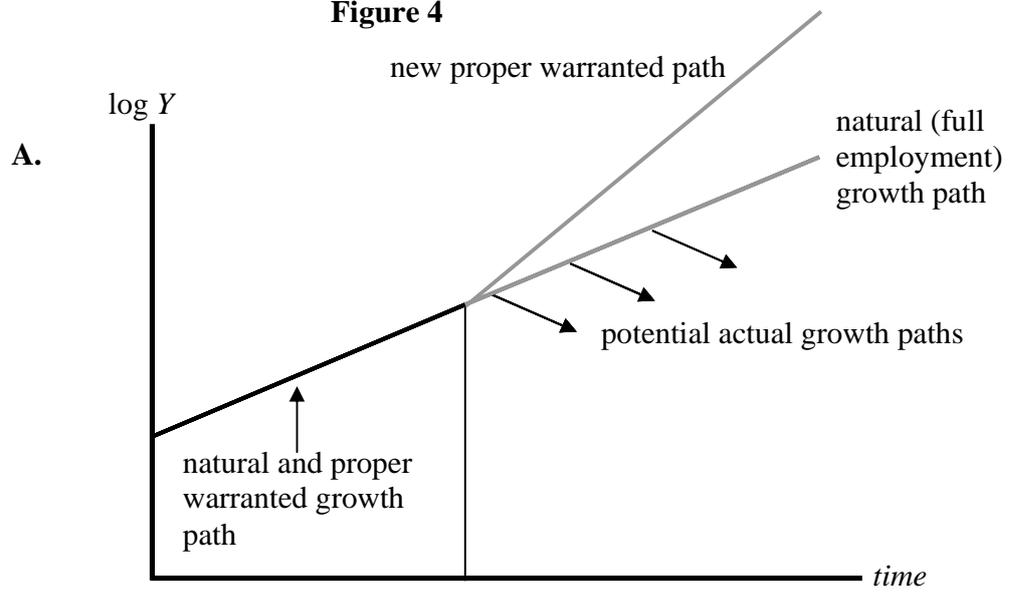
Figure 5
Does a high warranted rate promote recession?

$Gap = -5.9(G_N - G_{PW}) + 16.1$
$R^2 = 0.40$

Output gap (GDP as a percentage of potential GDP)

U.S. Data, 1930-2005
Source: See Appendix

Natural rate of growth (G_N) less the proper warranted rate of growth (G_{PW})
(percentage points)
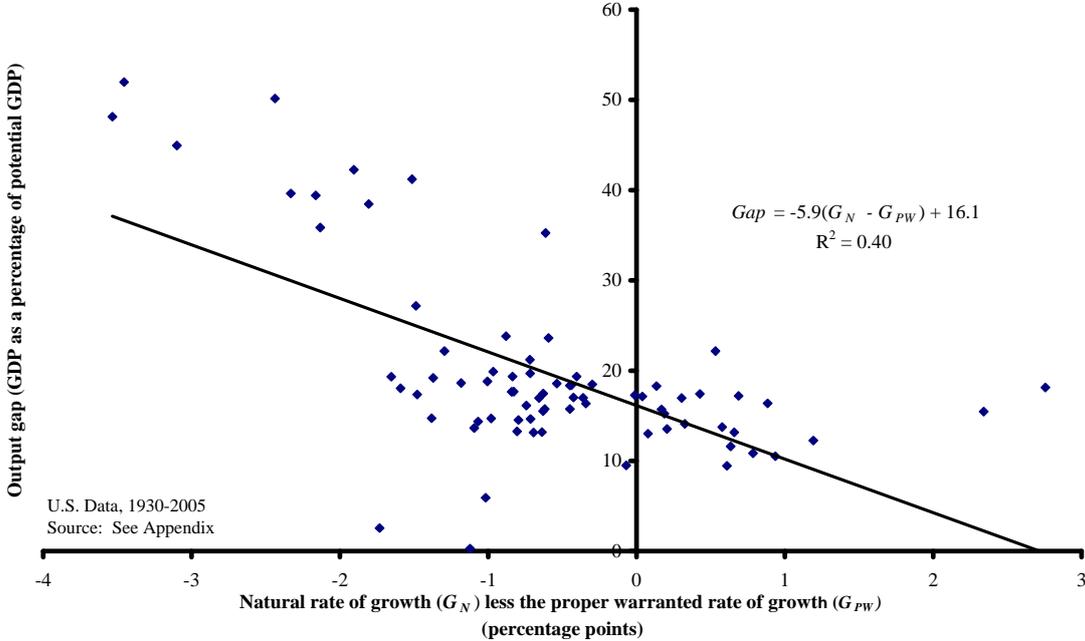
Figure 6.
Relative Citations of Harrod and Solow on Growth

Notes: Data are the number of articles in economics journals in the JSTOR Archive in which ("Harrod" and "growth") or ("Solow" and "growth"), scaled by the total number of articles and indexed so that the maximum value (177 for Solow in 1970) = 100. Harrod data run 1939-2005; Solow data, 1952-2005. Data were collected from 79 journals in JSTOR on 8 May 2008.

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